

SYNERGETICS AND THEORY OF CHAOS

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CONTROL OF CHAOS

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The idea of a “Combinatorial Chaotics” or Chaotic was as is well known, originally suggested by V.V. Gritsak-Groener in his pioneering article [1]. In this article we construct a control for combinatorial chaotic. The control of the flows in chaotic graphs have direct interpretation in terms of control combinatorial chaotic. The limiting cases are such representations follows straightforwardly. Furthermore, the degree of chaotic is determined for large classes of a combinatorial chaos. We also construct the computational algorithms of this the problems.

Key words: chaos, chaotic, algorithm, geotopia.

Мій друже, знаю я давно,
 Що скоро нас життя дістане...
 І серце в Землю загребуть
 І вже нічого не повстане.
 Коли зупинимся в путі,
 Або зануримся в туманах,
 Ти відпочить до мене йди,
 А я — до тебе, друг бажаний!

V. V. Gritsak-Groener “Псалом 1”

1. Glossary

1.1.

Let **A**, **B** be the sets. The map $\mathcal{F}: \mathbf{A} \longrightarrow 2^{\mathbf{B}}$ is a *multimap (mm)*

$$\mathcal{F}: \mathbf{A} \xrightarrow{\alpha} \mathbf{B}. \tag{1}$$

A is called an *image* for \mathcal{F} and **B** is called a *preimage* for \mathcal{F} . The multimap \mathcal{F} is said to be a *simple multimap* if $|\mathcal{F}(\alpha)| \leq 1$ for all $\alpha \in \mathbf{A}$. \mathcal{F} is called a *full* if $|\mathcal{F}(\alpha)| \neq \emptyset$ for all $\alpha \in \mathbf{A}$. The full **mm** \mathcal{F} is called a *injection* if $\alpha \neq \beta \Rightarrow \mathcal{F}(\alpha) \neq \mathcal{F}(\beta)$, where $\alpha, \beta \in \mathbf{A}$. Consider $\mathbf{A}_1 \subseteq \mathbf{A}$. The union

$$\mathcal{F}(\mathbf{A}_1) = \bigcup_{\alpha \in \mathbf{A}_1} \mathcal{F}(\alpha)$$

is called a *view* for \mathcal{F} . **mm** \mathcal{F} is a *surjection* if $\mathcal{F}(\mathbf{A}) = \mathbf{B}$.

Let $\mathcal{F}_1, \mathcal{F}_2, \otimes, \odot$ are a **mm** $\mathbf{A} \xrightarrow{\alpha} \mathbf{A}, \alpha \in \mathbf{A}$. By definition, put:

(i) $(\mathcal{F}_1 \cap \mathcal{F}_2)\alpha \stackrel{\text{def}}{=} \mathcal{F}_1(\alpha) \cap \mathcal{F}_2(\alpha);$

(ii) $(\mathcal{F}_1 \cup \mathcal{F}_2)\alpha \stackrel{\text{def}}{=} \mathcal{F}_1(\alpha) \cup \mathcal{F}_2(\alpha);$

(iii) $(\mathcal{F}_1 \mathcal{F}_2)\alpha \stackrel{\text{def}}{=} \mathcal{F}_1(\mathcal{F}_2(\alpha));$

(iv) $\otimes \alpha \stackrel{\text{def}}{=} \alpha;$

(v) $\odot \alpha \stackrel{\text{def}}{=} \emptyset.$

Let \mathcal{F} is the **mm** $\mathcal{F}: \mathbf{A} \xrightarrow{\alpha} \mathbf{A}$. The pair

$$\mathcal{G} = (\mathcal{F}, \mathbf{A}) \tag{2}$$

is called a *control graph* \mathcal{G} of **mm** \mathcal{F} . The elements of the set **A** are a *nodes* of \mathcal{G} .

The pairs $\mathbf{u} = (\alpha, \mathcal{A}(\alpha))$ are called an *arrows* of \mathcal{G} , where the α is a *tail* of \mathbf{u} and $\mathcal{A}(\alpha)$ is a *spike* of \mathbf{u} .

Let \mathbb{N} denote the set of natural numbers. For $\mathbf{n} \in \mathbb{N}$, put

$$\mathbb{N}(\mathbf{n}) = \{\mathbf{k} \in \mathbb{N} : \mathbf{k} \leq \mathbf{n}\}. \quad (3)$$

Suppose $\mathbf{mm} \ell : \mathbb{N}(\mathbf{n}) \rightarrow \mathbf{A}$ is a surjection. By \mathbf{A}_i denote $\ell(\mathbf{i})$, where $\mathbf{i} \in \mathbb{N}(\mathbf{n})$. An *indexed family of sets* \mathfrak{R} (or *indexed family* or *ifa*) is a pair

$$(\mathbb{N}(\mathbf{n}), \ell)$$

and is denoted by

$$\mathfrak{R} = (\mathbf{A}_i : \mathbf{i} \in \mathbb{N}(\mathbf{n})). \quad (4)$$

An indexed family \mathfrak{R} is called a *personal family* if $\mathbf{A}_i \neq \mathbf{A}_j$ when $\mathbf{i} \neq \mathbf{j}$. The personal family (3) is called a *partition* of the set \mathbf{A} if the following conditions hold:

- (i) $\mu(\mathbf{A}_i) \neq \emptyset$ when $\mathbf{i} \in \mathbb{N}(\mathbf{n})$;
- (ii) $\mathbf{A}_i \cap \mathbf{A}_j = \emptyset$ when $\mathbf{i}, \mathbf{j} \in \mathbb{N}(\mathbf{n})$ and $\mathbf{i} \neq \mathbf{j}$;
- (iii) $\bigcup_{1 \leq i \leq n} \mathbf{A}_i = \mathbf{A}$.

\mathfrak{R} (3) is called a *structure* if the following conditions hold:

- (a) $\bigcup_{1 \leq i \leq n_0, 1 \leq n_0 \leq n} \mathbf{A}_i \in \mathfrak{R}$;
- (b) $\bigcup_{1 \leq i \leq n_0, 1 \leq n_0 \leq n} \mathbf{A}_i \in \mathfrak{R}$.

By definition put $\mathbf{A}_i^c = \mathbf{A} \setminus \mathbf{A}_i$. A family sets $\mathcal{S}^c = \{\mathbf{A}_i^c : \mathbf{i} \in \mathbb{N}(\mathbf{n})\}$ is called a *complement of the ifa* $\mathcal{S} = \{\mathbf{A}_i : \mathbf{i} \in \mathbb{N}(\mathbf{n})\}$. A *ifa* $\mathcal{S} = \{\mathbf{A}_i : \mathbf{i} \in \mathbb{N}(\mathbf{n})\}$ is called a *self-complement* if from $\mathbf{A}_i \in \mathcal{S}$ it follows that $\mathbf{A}_i^c = \mathbf{A} \setminus \mathbf{A}_i = \mathbf{A}_i \in \mathcal{S}$.

Suppose

$$\mathcal{H} = (\mathbf{A} : \mathbf{C}_i, \mathbf{i} \in \mathbb{N}(\mathbf{n}))$$

is an indexed family such that

- (1) $\mu(\mathbf{C}_i) \neq \emptyset$,
- (2) if $\mathbf{C}_i \subseteq \mathbf{C}_j \Rightarrow \mathbf{C}_i = \mathbf{C}_j$ when $\mathbf{i} \neq \mathbf{j}$.

We call \mathcal{H} a *chaotic* on the set \mathbf{A} , where see figure 1.

A chaotic \mathcal{H} on the set \mathbf{A} is the chaotic of *circuits* of a *matroid* \mathcal{M} (see (5)) on the set \mathbf{A} if $\emptyset \notin \mathcal{C}$ and \mathcal{C} satisfies the *elimination axiom* :

(ax) whenever $\mathbf{Z}^1 \neq \mathbf{Z}^2 \in \mathcal{C}$ and $\mathbf{A} \ni \alpha \in \mathbf{Z}^1 \cap \mathbf{Z}^2$, there is a $\mathbf{Z}^0 \in \mathcal{C}$ with $\mathbf{Z}^0 \subseteq \mathbf{Z}^1 \cup \mathbf{Z}^2 \setminus \{\alpha\}$.

$$\mathcal{M} = (\mathcal{C} = \{\mathbf{Z}_i\} : \mathbf{i} \in \mathbb{N}(\mathbf{n})) \quad (6)$$

1.2.

Let \mathbf{A}, \mathbf{B} be a sets and $\mathcal{F} : \mathbf{A} \rightarrow \mathbf{B}$ is a multimap, $\emptyset \neq \mathbf{d} \subseteq \mathbf{B}$.

Consider a set $\mathcal{F}^*(\mathbf{d}) = \{\alpha \in \mathbf{A} \mid \mathcal{F}(\alpha) \cap \mathbf{d} \neq \emptyset\}$. By symbol $\mathcal{F}^*(\mathbf{d})$ denote $\{\alpha \in \mathbf{A} \mid \mathcal{F}(\alpha) \subseteq \mathbf{d}, \mathcal{F}(\alpha) \neq \emptyset\}$. Suppose $\mathbf{d} = \emptyset$; then $\mathcal{F}^*(\mathbf{d}) = \mathcal{F}^*(\mathbf{d}) = \emptyset$. $\mathcal{F}^*(\mathbf{d})$ is an *upper inverse multimap* and $\mathcal{F}^*(\mathbf{d})$ is a *lower inverse multimap*. Similarly, $\mathcal{F}^*(\mathbf{d}) : \mathbf{B} \rightarrow \mathbf{A}$, $\mathcal{F}^*(\mathbf{d}) : \mathbf{B} \rightarrow \mathbf{A}$ are the multimap. By definition, we have $\mathcal{F}^*(\mathbf{d}) \subseteq \mathcal{F}^*(\mathbf{d})$. The subset \mathbf{d} is called a *dense* if $\mathcal{F}^*(\mathbf{d}) = \mathcal{F}^*(\mathbf{d})$.

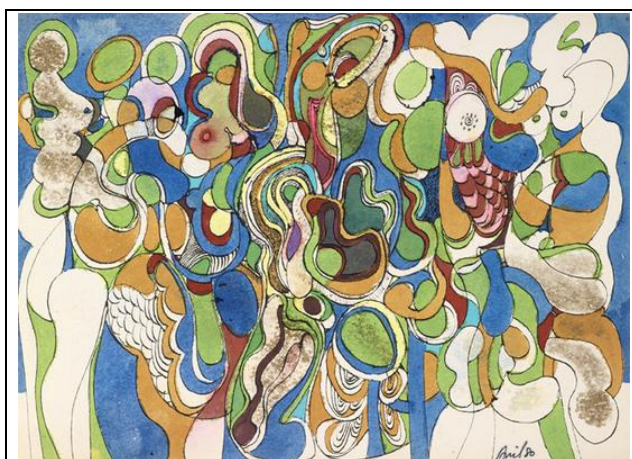


Fig. 1. Dmytro Pollack. On the same topic.(5)

Lemma 1. Suppose $\mathcal{F}: A \dashrightarrow B$ be the full multimap, $r \subseteq A$, $d, d_1, d_2 \subseteq B$; then the following conditions are fulfilled :

- (1) $r \subseteq \mathcal{F}^* \mathcal{F}(r)$;
- (2) $d \supseteq \mathcal{F} \mathcal{F}^*(d)$;
- (3) $d \cap \mathcal{F}(A) \subseteq \mathcal{F} \mathcal{F}^*(d)$;
- (4) $r \subseteq \mathcal{F} \mathcal{F}^*(r)$;
- (5) $(\mathcal{F}^*(d))^c = \mathcal{F}^*(d^c)$;
- (6) $\mathcal{F}^*(d)^c = \mathcal{F}^*(d^c)$;
- (7) $\mathcal{F}^*(d_1 \cup d_2) = \mathcal{F}^*(d_1) \cup \mathcal{F}^*(d_2)$;
- (8) $\mathcal{F}^*(d_1 \cup d_2) \supseteq \mathcal{F}^*(d_1) \cup \mathcal{F}^*(d_2)$.

Proof. This lemma can be proved by direct calculations.

1.3.

A suggestive way of writing “ α is preferred to β ” is “ $\alpha \succ \beta$ ”. Then it is to define “ $\beta \succ \alpha$ ” by “ $\alpha \preccurlyeq \beta$ ” and “ $\alpha \sim \beta$ ” by “ $\alpha \succ \beta$ ”, “ $\beta \succ \alpha$ ”. “ $\alpha \succ \beta$ ” by “ $\alpha \succ \beta$ ” but not “ $\alpha \sim \beta$ ”. A binary



Fig. 2. Anatoly Fomenko. The inverse multimap

relation (on A) is a subset \succ of $A \times A$. Frequently we write “ $\alpha \succ \beta$ ” $\iff (\alpha, \beta) \in \succ$ when $\alpha, \beta \in A$. If $\alpha \succ \beta$, then we shall say α is preferred to β . A binary relation on A is called

- 1) reflexive if $\alpha \succ \alpha$,
- 2) complete if $\alpha \succ \beta$ or $\beta \succ \alpha$ holds true for any $(\alpha, \beta) \in A \times A$,
- 3) transitive if $\alpha \succ \beta, \beta \succ \gamma$ implies $\alpha \succ \gamma$ ($\alpha, \beta, \gamma \in A$),
- 4) symmetric if $\alpha \succ \beta$ implies $\beta \succ \alpha$,
- 5) antisymmetric if $\alpha \succ \beta, \beta \succ \alpha$ implies $\alpha = \beta$,
- 6) asymmetric if $\alpha \succ \beta$ implies that $\beta \succ \alpha$ does not hold true.

A binary relation \succ on A is called a preference if \succ reflexive, transitive, and complete.

Let \succ be a binary relation on A . Then an acute hull $\succ \gg \succ$ of \iff there exists a sequence $\alpha = \alpha_0, \dots, \alpha_n = \beta$ such that $\alpha_i \succ \alpha_{i+1}$ ($i \in \mathbb{N}(n-1)$).

For every fixed $\alpha^* \in A$ let $\mathfrak{S}(\succ, \alpha^*) = \{\alpha \in A : \alpha^* \succ \alpha\}$. Similarly, $\mathfrak{S}(\succ, \alpha^*) = \{\alpha \in A : \alpha^* \succ \alpha\}$.

Let U is a finite set. A digraph D is a pair is a pair $D = (U, \succ)$. A ditree dT is a digraph (U, \succ) such that there exist an element $\alpha^0 \in U$ (to be called a root of the digraph) having the following properties:

- a. $\alpha \gg \alpha^0$ ($\alpha \in U$),
- b. $\mathfrak{S}(\succ, \alpha^0) = \emptyset$,
- c. $\mu(\mathfrak{S}(\succ, \alpha)) = 1$ ($\alpha \neq \alpha^0$).

The elements of the set U are a vertex of dT . The pairs $u = (\alpha, \beta)$ are called an arrows of dT if $\mathfrak{S}(\succ, \alpha) = \beta$.

2. Control

2.1.

Let \mathbf{A} be the finite sets.
 We shall say that the chaotic

$$\mathbb{H} = (\mathbf{A} : \mathbf{C}_i, i \in \mathbb{N}(n)) \tag{7}$$

is a *controlled chaos* \mathbb{H} , where $[i] \in \mathbb{N}(n)$ is a *controller*, the index family $(\mathbf{C}_i, i \in \mathbb{N}(n))$ are a *territory of the controller* $[i]$, and $\mathbf{Z} = \{[1], \dots, [n]\}$ is a *control-brigade* (or *brigade*). The elements of set \mathbf{A} are a *position of control* for chaos \mathbb{H} , \mathbf{A} is a *position-set*.

Suppose the pair $(\mathbf{Z}^1, \mathbf{Z}^2)$ is partition \mathbf{Z} when \mathbf{Z}^1 are an *active controllers*, and \mathbf{Z}^2 are an *passive controllers*. We shall say that for the chaotic \mathbb{H} there exists a control if the following conditions hold:

(a) we have a multimap

$$\Omega : \mathbf{A} \multimap \mathbf{A}, \tag{8}$$

then this is called a law of the control;

(b) for any $[i]$ there exists a preferences \succsim_i , then this \succsim_i is called a *preference* of controller $[i]$.

Let $\mathbf{C}_0 \stackrel{def}{=} (\alpha : \Omega(\alpha) = \emptyset)$ and using a transformation of Ω we get $\Omega(\mathbf{C}_0) \cap \mathbf{C}_k = \emptyset$, where $k \in \mathbb{N}(n)$.

Suppose $\alpha_0 \in \mathbf{A}$ be a *beginning* element of position. We shall say that a *brigade* $\mathbf{Z} = \{[1], \dots, [n]\}$ *exerts control* over the chaotic \mathbb{H} if the following steps hold:

- (1) let $([i_1], \dots, [j_1], \dots, [l_1]) \neq \emptyset$ ($1 \leq i_1 \leq \dots \leq j_1 \leq \dots \leq l_1 \leq n$) is maximum allowable of controller number such that $\Omega(\alpha_0) \cap \mathbf{C}_{t_1} \neq \emptyset$, where $t_1 \in (i_1, \dots, j_1, \dots, l_1)$ whence the controller $[t_1]$ *choose* element of position $\alpha_1^{t_1} \in \Omega(\alpha_0)$, control is continue and we have the controllable positions $\alpha_0, \alpha_1^{i_1}, \dots, \alpha_1^{j_1}, \dots, \alpha_1^{l_1}$;
- (2) if $([i_1], \dots, [j_1], \dots, [l_1]) = \emptyset$ the control is *finished*;
- (3) let $t_1 \in (i_1, \dots, j_1, \dots, l_1)$, if $([i_{r(t_1)}], \dots, [j_{r(t_1)}], \dots, [l_{r(t_1)}], \dots, [l_{r(t_1)}]) \neq \emptyset$ is maximum allowable of controller number such that $\Omega(\alpha_1^{t_1}) \cap \mathbf{C}_{t_2} \neq \emptyset$, where $t_2 \in (i_{r(t_1)}, \dots, j_{r(t_1)}, \dots, j_{r(t_1)}, \dots, l_{r(t_1)}, \dots, l_{r(t_1)})$ whence the controller $[t_2]$ *choose* element of position $\alpha_2^{t_2} \in \Omega(\alpha_1^{t_1})$, control is continue and we have the controllable positions

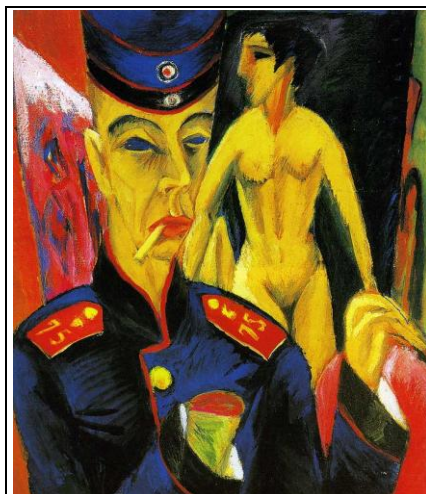
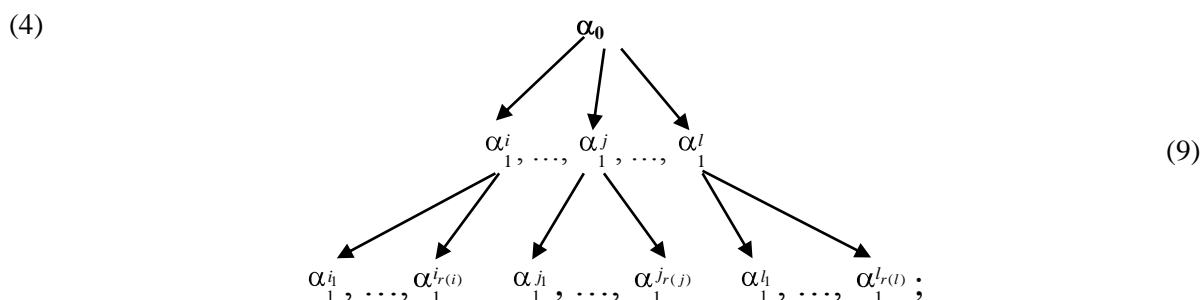


Figure 3. Fritz Grosz. Controller



- (1) if $([i_{r(t_1)}], \dots, [i_{r(t_1)}], \dots, [j_{r(t_1)}], \dots, [j_{r(t_1)}], \dots, [l_{r(t_1)}], \dots, [l_{r(t_1)}]) = \emptyset$ the control is *finished*;
- (2) and so on, as so long the controller induce the nonempty positions in the ditree $dT = (U, \succ)$ (9).

A ditree dT of sampling is a oriented graph $\Gamma = (\mathbf{V}(\Gamma), \mathbf{E}(\Gamma))$, where $\mathbf{V}(\Gamma)$ is a set *vertex* for Γ and $\mathbf{E}(\Gamma)$ is a set *arrows* for Γ . A *control number* of a control is, by definition, the number $\mu(\mathbf{V}(\Gamma))$ of vertex for Γ and a *capacity* of a control is the number $\mu(\mathbf{E}(\Gamma))$.

2.2.

A preference \succsim_i of controller $[\mathbf{i}]$ it is possibility represent such that a numerical bounded function

$$f_i(x): x \longrightarrow \mathbb{R}$$

as follows:

$$\alpha \succsim_i \beta \Leftrightarrow f_i(\alpha) \geq f_i(\beta). \quad (10)$$

Then we shall say that a brigade $\mathbf{Z} = \{[\mathbf{1}], \dots, [\mathbf{n}]\}$ exerts control with penalty function $\mathbf{f}_i(\mathbf{x})$ over the chaotic \mathbf{H} .

Commentary. Surprisingly, penalty function das not always happen that exist.

If a controller $[\mathbf{i}]$ is the active, it is customary were more preferable to position of control with respect to \succsim_i , $f_i^+(\mathbf{C}_i) = \sup_{x \in C_i} \mathbf{f}_i(\mathbf{x})$ will be written in terminology of penalty function.

If a controller $[\mathbf{i}]$ is the passive, it is not customary were less preferable to position of control with respect to \succsim_i , $f_i^-(\mathbf{C}_i) = \inf_{x \in C_i} \mathbf{f}_i(\mathbf{x})$ will be written in terminology of penalty function.

3. Chaotic

Let \mathbf{A} be the finite sets. A combinatorial chaotic (or simply chaotic) on \mathbf{A} is a family \mathbf{C} of subsets of \mathbf{A} , such that no element of \mathbf{C} is contained in any other. A chaotic is conveniently expressed by:

$$\mathbf{H} = (\mathbf{A} : C_i, i \in \mathbb{N}(n)), \quad (11)$$

where C_i is a cycle of chaotic \mathbf{H} , and the index family

$$\mathbf{C} = (C_i, i \in \mathbb{N}(n))$$

are a territory of the controller $[\mathbf{i}]$, and a family sets

$$\mathbf{C}^c = (C_i^c, i \in \mathbb{N}(n))$$

is a complement of the family sets \mathbf{C} .

3.1.

A set $(\bar{B})_{\mathbf{H}} \subseteq \mathbf{A}$ is called a *closure* of a subset $B \subseteq \mathbf{A}$ if the following conditions hold:

- (i) $B \subseteq (\bar{B})_{\mathbf{H}}$;
- (ii) $\beta \in (\bar{B})_{\mathbf{H}} \setminus B \Leftrightarrow$ there exists two sequences $\{\alpha_t\}, \{U_t\}$, where $\alpha_t \in \mathbf{A}$, $U_t \in \mathbf{C}$, $\alpha_t \neq \alpha_k$, $\alpha_n = \beta$, $U_t \neq U_k$ such that $\alpha_t \in U_t \subseteq B \cup (\bigcup_{j=1, t}^n \alpha_j)$, where $1 \leq t < k \leq n$.

We write it \bar{B} for short.

A subset $\mathbf{D} \subseteq \mathbf{A}$ is called a *flat* (or a *closed* set of \mathbf{H}) if $\bar{D} = \mathbf{D}$. A flat $\mathbf{P} \subseteq \mathbf{A}$, $\mu(\mathbf{P}) = \mathbf{1}$ is called a *loop* \Leftrightarrow the element $\alpha \in \mathbf{A}$ is a *loop* if $\alpha \notin C_i$ for $\forall C_i \in \mathbf{C}$ and the element β is a *coloop* if $\beta \in C_i$ for $C_i \in \mathbf{C}$. The minimal flat subsets $\mathbf{R} \subseteq \mathbf{A}$ of are called an *atoms*.

An *envelope* set of A is every $\mathfrak{Z}(D) \subseteq A$ such that $\mathfrak{Z}(D) = \overline{D}/D$.

The definition of closure implies easily the following property:

1. For every $D \subseteq A$ $D \subseteq \overline{D}$ and $\overline{\overline{D}} = \overline{D}$.
2. $D_1 \subseteq D_2 \Rightarrow \overline{D_1} \subseteq \overline{D_2}$ for all $\overline{D_1}, \overline{D_2}$.
3. If $D = \overline{D \cup \alpha} / (\overline{D} \cup \alpha) \neq \emptyset$ then there is $\beta \in D$ such that $\overline{D \cup \alpha} = \overline{D \cup \beta}$.

Remark: “there is” can be changed to “for all” $\Leftrightarrow \mathbf{H}$ is a matroid.

$\mathbf{D} \subseteq \mathbf{A}$ is a *spanning* set of the chaotic \mathbf{H} if $\overline{\mathbf{D}}$. A subset $\mathbf{N} \subseteq \mathbf{A}$ is called *independent* in \mathbf{H} if $\forall \mathbf{E} \in \mathbf{C}, \mathbf{E} \setminus \mathbf{N} \neq \emptyset$. Minimal spanning subsets $\mathbf{B} \subseteq \mathbf{A}$ are *bases* of \mathbf{H} . A maximal nonspanning flat subset $\mathbf{K} \subseteq \mathbf{A}$ is a *coatom* of \mathbf{H} .

3.2.

Let $\mathbf{H} = (\mathbf{A} : \mathbf{C}_i, i \in \mathbf{N}(n))$ be a chaotic, $\alpha \in \mathbf{A}$ be an element of \mathbf{H} , $\mathbf{A}_0 \subseteq \mathbf{A}$.

Theorem 1. $\mathbf{H}(\mathbf{A}_0) = \{ \mathbf{A}_0 : \mathbf{C}_i(\mathbf{A}_0), i \in \mathbf{N}(n) \}$ is chaotic, where $\mathbf{C}_i(\mathbf{A}_0) = \mathbf{C}_i \cap \mathbf{A}_0 \neq \emptyset$.

Theorem 2. If $\mathbf{H} \setminus \alpha = \{ \mathbf{A} \setminus \{ \alpha \}, \mathbf{D} : \alpha \notin \mathbf{D} \in \mathbf{C} \}$ such that α is not a coloop of \mathbf{H} or $\mathbf{H} \setminus \alpha = \{ \mathbf{A} \setminus \{ \alpha \}, \mathbf{D} \setminus \{ \alpha \} : \mathbf{D} \in \mathbf{C} \}$ such that α is a coloop of \mathbf{H} , then $\mathbf{H} \setminus \alpha$ is chaotic obtained by *separation* of α .

Theorem 3. If $\mathbf{H} / \alpha = \{ \mathbf{A} \setminus \{ \alpha \}, \mathbf{D} \setminus \{ \alpha \} : \alpha \in \mathbf{D} \in \mathbf{C} \}$ such that α is not a loop of \mathbf{H} or $\mathbf{H} / \alpha = \{ \mathbf{A} \setminus \{ \alpha \}, \mathbf{D} : \mathbf{D} \in \mathbf{C} \}$ such that α is a loop of \mathbf{H} , then \mathbf{H} / α is chaotic obtained by *detachment* of α .

For the proofs we refer the reader in [1].

A minor of \mathbf{H} is a chaotic $\mathbf{m-H}$ that can be obtained from \mathbf{H} by a sequence of \mathbf{m} separations and detachments.

3.3.

A chaotic \mathbf{H} is an *atomistic* if for every a close set $\mathbf{B} \subseteq \mathbf{A}$ there exists the sets of atoms $\mathbf{R}_j \subseteq \mathbf{A}, j \in \mathbf{J}$ such that

$$\overline{\bigcup_{j \in \mathbf{J}} \mathbf{R}_j} = \mathbf{B}$$

We may assume that an *julies* [1] are model of granular chaos and an *anthills* [1] are model of uniformly chaos. These results can be summarized as follows.

Theorem 4. Let a chaotic \mathbf{H} be a jula. If \mathbf{H} is have not a loops, then

$$\mu(\mathbf{D}) \geq 3$$

of all cycle $\mathbf{D} \in \mathbf{C}$.

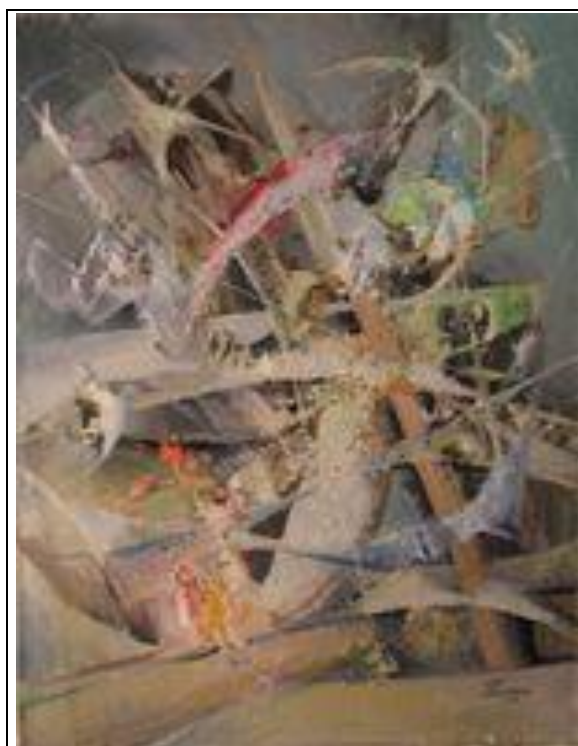


Figure 4. Ivan Nevidomyj. Chaotic and (13)

Corollary 4.1. There exists a polynomial complex algorithm of to see that \mathbf{H} be a jula.

Theorem 5. Let a chaotic $\mathbf{H} = (\mathbf{A} : \mathbf{C}_i, i \in \mathbf{N}(\mathbf{n}))$ be an anthill, $\mathbf{B} \subseteq \mathbf{A}$ be an arbitrary independence subset of \mathbf{H} . Then there exists an element $\beta \in \mathbf{B}$ such that

$$\beta \notin (\mathbf{B} / \{\beta\})^c \tag{14}$$

Corollary 5.1. There exists a polynomial complex algorithm of to see that \mathbf{H} be an anthill.

An the chaotic \mathbf{H} is a *quasimatroid* if \mathbf{H} is both a jula and an anthill, see in figure 5.

3.4.

Let $\mathfrak{S} = (\mathbf{A}_1, \mathbf{C})$ be a chaotic. By $\dimh(\mathfrak{S}) \in \mathbf{Z}^+$ denote a degree of disorder of a chaotic \mathfrak{S} . If \mathfrak{S} is a matroid, then $\dimh(\mathfrak{S}) = 0$. Let $\mathfrak{S}^* = (\mathbf{A}^*, \mathbf{C}^*)$ is an other a chaotic. Further, let $\alpha \in \mathbf{A}_1$ and $\alpha_1 \in \mathbf{A}^*$. We can assume that

$$|\dimh(\mathfrak{S}) - \dimh(\mathfrak{S}^*)| = 1 \tag{15}$$

if the following conditions hold:

- 1) $\mathfrak{S} = \mathfrak{S}^* \setminus \alpha_1$,
 - 2) $\mathfrak{S} = \mathfrak{S}^* / \alpha_1$,
 - 3) $\mathfrak{S}^* = \mathfrak{S} \setminus \alpha$,
 - 4) $\mathfrak{S}^* = \mathfrak{S} / \alpha$.
- (15)*

The chaotics \mathfrak{S} and \mathfrak{S}^* are called *adjacent*. A *x-walk* is a sequence \mathbf{L} :

$$\mathfrak{S}_1, \mathfrak{S}_2, \dots, \mathfrak{S}_m \tag{16}$$

of the chaotic $\mathfrak{S}_j, j = [1, m]$, in which \mathfrak{S}_j and \mathfrak{S}_{j+1} are adjacent, \mathfrak{S}_1 is the input of \mathbf{L} and \mathfrak{S}_m is the outcome of \mathbf{L} .

Theorem 6. Let $\mathbf{H} = (\mathbf{A}, \mathbf{C}), \mathbf{A} \neq \emptyset, \mathbf{H}_1 = (\mathbf{A}_1, \{(1,2), (1,3)\}), \mathbf{H}_2 = (\mathbf{A}_2, \{(1, 2, 3)\})$ are a chaotics. A chaotic \mathbf{H} is a matroid $\Leftrightarrow \mathbf{H}$ has not be *x-walk* $\mathbf{L} = \mathfrak{S}_1, \mathfrak{S}_2, \dots, \mathfrak{S}_m$ (16), where $\mathfrak{S}_1 = \mathbf{H}$ and $\mathfrak{S}_m = \mathbf{H}_1$ or $\mathfrak{S}_m = \mathbf{H}_2$.

Corollary 6.1. Suppose $\mathbf{H} = (\mathbf{A}, \mathbf{C}), 0 < \mu(\mathbf{A}) < \infty$ be a chaotic, then there exists a polynomial complex algorithm of to see that \mathbf{H} be a matroid.

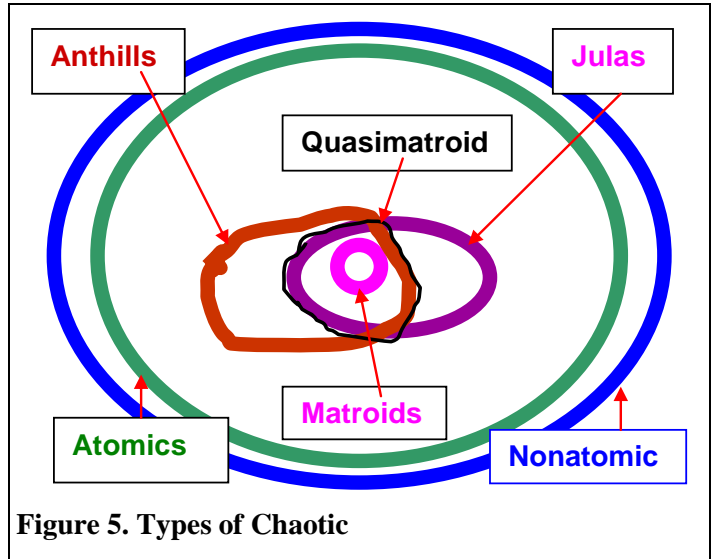


Figure 5. Types of Chaotic

4. Control of Chaotic

4.1.

Many problems in physics, biology and medicine involve determining the measure of disorder of several objects. For instance, a finding that a structure which computed a degree of chaotic can approach determining the disorderly structure of swelling or computed a degree of chaotic of the geotectonic structures. The figure 6 shows the real geotectonic structure consisting of three bands of varying degrees of randomness.

As is known, see [Calculate], voltage pulses are transmitted to the tectonic structures, only in certain ranges of the degree of chaotic. Therefore, it is necessary to control the degree of randomness in order to prevent the consequences of an earthquake or transmission voltage pulses from the outside.



Fig. 6. Image showing three types of the geotectonic texture. Mülhausen, Switzerland

4.2.

Consider the algorithm for solving the problem of controlling the degree of chaotic.

Let \mathbf{A} be the finite sets, $\mathbf{A}_0 \subseteq \mathbf{A}$, the chaotic $\mathbf{H} = (\mathbf{A} : \mathbf{C}_i, \mathbf{i} \in \mathbf{N}(\mathbf{n}))$ is the controlled chaos and the chaotic $\mathbf{H}(\mathbf{A}_0) = \{ \mathbf{A}_0 : \mathbf{C}_i(\mathbf{A}_0), \mathbf{i} \in \mathbf{N}(\mathbf{n}) \}$ is the position of a *beginning control*, $\mathbf{Z} = \{[\mathbf{1}], \dots, [\mathbf{n}]\}$ is a brigade of active controlled.

We control the chaotic \mathbf{H} were taken control sequence from $\mathbf{1}$ to \mathbf{n} if the following conditions hold:

(i) $\mathbf{B}_1 = \mathbf{A}_0$;

(ii) suppose $\mathbf{C}_i(\mathbf{A}_0) \neq \emptyset$; then (iii); if $\mathbf{C}_i(\mathbf{A}_0) = \emptyset$; then (ii);

(iii) suppose exists $\{ \alpha_i \}, \{ \mathbf{C}_i(\mathbf{A}_0) \}$, where $\alpha_i \in \mathbf{C}_i(\mathbf{A}_0)$ such that $\mathbf{C}_i(\mathbf{A}_0) \subseteq \mathbf{B}_{i-1} \cup \alpha_i$ and $\mathbf{C}_i(\mathbf{A}_0) \not\subseteq \mathbf{B}_{i-1}$, $\mathbf{B}_i = \mathbf{B}_{i-1}$; then (ii).

Theorem 7. $\mathbf{B}_n = \overline{\mathbf{A}_0}$.

Algorithm 1 builds a closure of a set in chaotic.

Theorem 8. If $\mathbf{H}(\overline{\mathbf{A}_0}) = \mathbf{H}$, then $|\dim(\mathbf{H}) - \dim(\mathbf{H}(\overline{\mathbf{A}_0}))| = 0$.

Further if the chaotic $\mathbf{H}(\overline{\mathbf{A}_0}) = \mathbf{H}$, then we connect the four passive controller

$$\mathbf{Z} := \mathbf{Z} \cup \{ \mathbf{J}, \mathbf{An}, \mathbf{Q}, \mathbf{M} \},$$

where a controller \mathbf{J} verify “the chaotic $\mathbf{H}(\overline{\mathbf{A}_0})$ is a jula“, a controller \mathbf{An} verify “the chaotic $\mathbf{H}(\overline{\mathbf{A}_0})$ is a anthill“, a controller \mathbf{Q} verify “the chaotic $\mathbf{H}(\overline{\mathbf{A}_0})$ is a quasimatroid“, a controller \mathbf{M} verify “the chaotic $\mathbf{H}(\overline{\mathbf{A}_0})$ is a matroid“.

Finally if the chaotic $\mathbf{H}(\overline{\mathbf{A}_0}) \neq \mathbf{H}$, then we connect the four active controller

$$\mathbf{Z} := \mathbf{Z} \cup \{ [\mathbf{I}], [\mathbf{II}], [\mathbf{III}], [\mathbf{IV}] \},$$

where a controller [I] construct the chaotic $H(\overline{A_0}) \setminus \alpha_1$, a controller [II] construct the chaotic $H(\overline{A_0}) \setminus \alpha$, a controller [III] construct the chaotic $H(\overline{A_0}) / \alpha_1$, a controller [IV] construct the chaotic $H(\overline{A_0}) / \alpha$, see (15) –(15)*. The brigade {[I], [II], [III], [IV]} check for exists the sequence $\mathfrak{S}_1, \mathfrak{S}_2, \dots, \mathfrak{S}_m$ (16), where $\mathfrak{S}_m = \{A_1, (1,2), (1,3)\}$ or $\mathfrak{S}_m = (A_2, \{(1, 2, 3)\})$. Thence

$$\dimh(H) \geq \dimh(H(A_0)) + m + 1. \quad (17)$$

If $\mathfrak{S}_m = \{A_3, (1,2), (3,4)\}$ or $\mathfrak{S}_m = \{A_4, (1,2)\}$

Thence

$$\dimh(H) = \dimh(H(A_0)) + m + 1. \quad (18)$$

4.3. Algorithm "Closet"

Listing procedures in pseudocode is as follows:

```

Input: Om
find:
nom — number of cycles;
A — elements of A-set (union of all cycles);
nA — number of elements of A-set;
T = Om (so all circuits coincide with cycles);
nmax = min(nom, nA) — (maximum length of f-sequences);
for kom=1:nom — build the circuit each cycle;
find:
AmOm = A \ Om(kom) - additions to the current cycle;
nAmOm - number of elements complement AmOm;
for knAmOm=1:nAmOm - check each item;
find:
b = AmOm(knAmOm) - an item that is checked;
for n=1:nmax - build f-sequence of length n;
find:
ap - all placement of length n for a(i); This is a two-dimensional array, its
every row is one placement of nA elements by n;
omp - all placement of length n for τ(i); This is a two-dimensional array, its
every row is one placement nom elements by n;
for ia = (all rows of the ap array) - begin to check the definition of closing;
find:
a - sequence a: Picks of the elements from the main A set with the numbers that are given
in the row number ia from the ap array;
if a(n)=b - first test whether the same last element of the sequence with b?
for iom=1:(all rows of the omp array) - in this case, keep checking;
find:
ntau - numbers of τ-sequences: row number iom array omp;
ClipYes = True - to check
for i=1:n - first test entry
if a(i) ∉ Om(ntau(i)),
ClipYes = False - not suitable
break - exit from the cycle "for i";
else - a(i) ∉ τ(i) - continue the inspection;
find:
B = Om(kom) ∪ (∪ a(j)) :
B = Om(kom) - ask the original B;
for j=1:i,
B = B ∪ a(j);
end - set B = Om(kom) ∪ (∪ a(j)) has been created
if Om(ntau(i)) ∉ B
ClipYes = False - not suitable
break - exit from the cycle "for i";

```

```
end
if ClipYes = True - everything fits
  T (kom) = T(kom) ∪ a - attach element to circuit
  break; leave the cycle for iom
end
```

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Послесловие автора:

Наиболее адекватны нашей модели геотектонические структуры. Они, в зависимости от степени их хаотичности (и совершенно вне зависимости от их плотности и химического состава) могут проводить или не проводить сигналы, волны плотности. Это порождает у любителей глубокой подземной связи искушение использовать наш прекрасный, уютный и не такой уж большой земной шарик в качестве телеграфа для посылки сообщений, не всегда с приятных. Именно на этой почве появились мои книги с моими дорогими профессорами Александром Сергеевичем Давыдовым и Георгием Трофимовичем Продайвода. Мы всегда были полны понимания рискованности развития этой темы. Но если Господь доверил именно нашим головам и рукам эту тему, мы сделаем это.

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Управление хаосом

Продолжается серия изучения структур хаоса. С этой работы мы начинаем не только наблюдать хаотические структуры и регистрировать их степень их хаотичности, но и управлять ими — в наши работы по хаосу входит динамика. Управление потоками в хаотических графиках имеет прямую интерпретацию в терминах комбинаторного управления хаотиками. Предельным случаем является прямолинейное представление. Наиболее адекватны нашей модели геотектонические структуры. Также построены вычислительные алгоритмы этой проблемы.

Ключевые слова: хаос, хаотик, алгоритм, геотектонические структуры.