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CLASSICAL MATHEMATICAL SOCIOMETRY. PART II.
MATHEMATICS, POETRY, PICTURE

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The article is a direct extension of the article by the same authors under the name “Classical mathematical sociometry. Part I”. The article is devoted to the axiomatization of social relations. Separately stand out classic axioms. Traditional mathematical glossary complements readers' knowledge of the elements of mathematical relations. Theorems 1 and 2 suggest 24 scenarios of possible social individual relations. The nonmonotonic triple spiral of life of human groups is based on three chains of waves charged to the final spot. The theory that we begin to explore in paragraph 5 is dual to the theory of ordering in sociometry, which we developed in our previous articles. It investigates not the ordering between persons, but the ordering of the decisions of one person (in the linear case) of his/her choices. As a matter of fact, this is exactly what happens during sociological polls, interrogations, questionnaires, psychiatric tests and other classical methods. We consider the mathematical theory of processing individual data collection, together with the axiomatics, the theorems, their proofs and mathematical implications.

Key words: relation, axiom, sociometry.

0. Introduction

Do we need attitudes, graphs, barons, dukes, chevaliers, princesses and kings to our Ukraine? Are the categories, principles, conscience necessary for the management of our state? These are the most important, fundamental questions of our book. But we will receive a complete answer to them only at the end of our book. First, we must go through the scrupulous and confused way of historical and mathematical research.

1. Relations

Throughout the book, simply the expression “relation” means a “binary relation”. We will mark the n -ratio, as we wrote earlier, separately.

The universal property of all relations V is the **equivalence** of

$$a_1 \succ a_2 \text{ and } a_2 \prec a_1,$$

in other words, it can be read from right to left and is equivalent to left-to-right. If V executes

$$a_2 \succ a_1 \text{ and } a_2 \prec a_1$$

at the same time, we will write

$$a_1 \sim a_2$$

and call a_1 is **equivalent** to a_2 for V , or just a_1 is equivalent to a_2 , when it does not cause ambiguity.

Further, if $a_1 \succ a_2$, but not $a_1 \sim a_2$, then we write $a_1 \triangleright a_2$, and we call a_1 as **strictly related** to a_2 in V . If for some elements a_1 and a_2 of the main set A on which the relation R is constructed, the pair $(a_1, a_2) \notin V$, in other words, does not execute $a_1 \succ a_2$, and not $a_2 \succ a_1$, then a_1 and a_2 will be called **incomparable** and denote $a_1 \nabla a_2$

Examples 1.

(a) Let P be the relation of belonging to one party, which in the socialist and communist parties is called the word “comrade”. We have, if Ivan is a friend of Peter, and Peter is a friend Ivan too. This means that in the party structures all relations are equivalent and there are no strict relations at all.

(b) Selection from set F of all possible political decisions such pairs (p_i, p_j) , where p_i the pre-

vious decision, and p_j the next so that as a result the program was solved or the intended purpose of the government or other management

(c) On the other hand, if we consider the relation F to the important example (b), then most of its elements, which are political decisions, or are strictly related to each other, or are at all incomparable. So, for solutions f_1 and f_2 we have $f_1 \succ f_2$, when solution f_1 must be performed necessarily after f_2 . If this is not the case, we have $f_1 \nabla f_2$.

2. Axioms of individual relations

Definition 1. The binary relations between members of society \mathfrak{R} can be classified by type depending on the following axioms:

- (1) $(a, a) \in \mathfrak{R}$ for $\forall a \in A$ is a **reflexivity**;
- (2) $(a, a) \notin \mathfrak{R}$ for $\forall a \in A$ is an **ireflexivity**;
- (3) $(a_1, a_2) \in \mathfrak{R} \Rightarrow (a_2, a_1) \in \mathfrak{R}$ for $\forall a_1, a_2 \in A$ is a **symmetry**;
- (4) $(a_1, a_2) \in \mathfrak{R} \Rightarrow (a_2, a_1) \notin \mathfrak{R}$ for $\forall a_1, a_2 \in A$ is an **isymmetry**;
- (5) $(a_1, a_2) \wedge (a_2, a_1) \in \mathfrak{R} \Rightarrow a_1 = a_2$ for $\forall a_1, a_2 \in A$ is an **ansymmetry**;
- (6) for $\forall a_1 \neq a_2 \in A \Rightarrow (a_1, a_2) \in \mathfrak{R} \vee (a_2, a_1) \in \mathfrak{R}$ is a **completeness**;
- (7) $(a_1, a_2) \wedge (a_2, a_3) \in \mathfrak{R} \Rightarrow (a_1, a_3) \in \mathfrak{R}$ for $\forall a_1, a_2, a_3 \in A$ is a **transitivity**;
- (8) for $\forall a_1, a_2 \in A \Rightarrow (a_1, a_2) \in \mathfrak{R} \vee (a_2, a_1) \in \mathfrak{R}$ is a **linear ordering**.

Commentary 1.

1. The relationship will be called the axiom of the order that is performed for them. For example, the relation \mathfrak{R} , for which the axiom of isymmetry is performed, is called an **isymmettic**.

2. In classical studies, besides ours, no more than 4 axioms are considered for individual decisions. But the classics did not read the classical book [1], or our book [2].

For a binary relation \mathfrak{R} one or more axioms of order may be performed. Thus, they are classified according to the order of the order in accordance with the table in Figure 1.



Picture 1. Vasyil Kandinsky. Order.

Order types \ Axioms	Qasi.	Free	Simply	Strongly simply	Strongly free	Part	Total	Equivalent
Reflexivity								
Transit.								
Symmetry								
Isymmetr.								
Ansymmet.								
Linier order								
Completeness								
Ireflexiity								

Figure 1.

Theorem 1.

Given the types of order, we have 22 axioms of individual relations.

Proof. The proof is simple, but voluminous and follows from figure 1. We throw out 42 absurd cases.

Commentary 2.

1. All 22 cases will be considered in detail, along with the algorithms for their calculation in the 'classic' and 'modern' series our articles devoted to sociometry.

2. The **n**-relations are also broken down into types that we will consider depending on the need.

Examples 2.

1. For example, for the order of the relationship between natural numbers “ \geq ” ($7 \geq 3$), the axioms of order — reflexivity, transitivity, antisymmetry, and strict completeness are fulfilled. So it has a type of linear order.

2. But, the relation **P** of examples will be irreflexive, symmetric, transitive and complete. There is no such type in the table in Figure 1, which means that the **P** ratio is not classical. Therefore, we have the full right to give him his name. We call the relation **P**, for which the axioms of reflexivity, symmetry, transitivity, and completeness are performed by a **party quasiorder**.

3. The ratio **F** of examples 1 will be only reflexive and asymmetric. As we see from Figure 1, **F** has a type of free order.

Theorem 2.

We have 24 ordering for classical theory of individual relations.

Proof. 23 ordering from figure 1 + the party quasiorder.

3. Important examples

Because of the utmost importance in econometrics, we will separately consider the type of binary relations of the **partial order** “ \leq ” and the **equivalence** “ \approx ”.

Definitions 2. The binary relation “ \leq ” on the groundset **A**

$$\leq \subseteq A \times A$$

is called a **part ordered (po)** on set **A**, if the following axioms are performed:

- (1) $x \leq x$ (reflexivity);
- (2) if $x \leq y$ and $y \leq x$, then $x = y$ (antisymmetry);
- (3) if $x \leq y$ and $y \leq z$, then (transitivity).

If also performed, the following axiom for “ \leq ”:

- (4) for any $x, y \in A$, $x \leq y$ or $y \leq x$ is performed,

then we call the partial order “ \leq ” a **total ordered (to)** on set **A**.

A nonempty set with **po** on it is called a **poset**. If the partial order is total, then the set together with the partial order is called a **chain**. Let **A** is a poset. $x, y \in A$. We use a notation $x < y$, and we call **x** a **strictly more (sm)** more than **y** if we perform simultaneously $x \leq y$ and $x \neq y$.

Examples 3.

1. A good example of partial order “ $<$ ” is the ratio of subordination in administrative structures, where $x < y$ is performed for some employees **x** and **y**, if **y** is the chief of **x**.

2. Somewhat more complicated po will be seniority in the army. After all, except for seniority $x \leq y$ for military **x** and **y**, if **y** is a commander of **x**, there are in the army, and there are also military titles that produce an additional seniority that may not coincide with the main commander.

3. The total order “ $<$ ” is most commonly encountered in objects expressed by numerical values. For example, a monetary units. So we have: 10 cents $<$ 10 dollars.

Definitions 3. The binary relation “ \approx ” on a groundset **A**

$$\approx \subseteq A \times A$$

is called a **equivalence** on the set **A** if the following axioms are fulfilled:

- (1) $x \approx x$ (reflexivity)
- (2) if $x \approx y$, then follows $y \approx x$ (symmetry)
- (3) if $x \approx y$ and $y \approx z$, then $x \approx z$ (traceability)

follows.
If $x \approx y$ is performed, then **x** and **y** are called equivalent, and the subset

$$A^{\approx}(x) = \{y \in A : x \approx y\} \subseteq A$$

is a **e class of equivalence**.

In general, for each equivalence relation “ \approx ” on a finite set **A**, it is always possible to select the elements $x_i \in A$, $i = [1, n]$, which are executed:

$$A = \bigcup_{i=1}^n A^{\approx}(x_i) \tag{3.1}$$

$$A^{\approx}(x_i) \cap A^{\approx}(x_j) = \emptyset \text{ for } i \neq j.$$

A set $A^{\approx}(x_i)$ is taken as an element $a^{\approx}(x_i)$, $i = [1, n]$, then set

$$A/\approx = \{a^{\approx}(x_i) : i = [1, n]\} \tag{3.2}$$

called a **factor-set** of the set **A** with respect to equivalence “ \approx ” and a factor-set is a very important construct in applications.

Examples 4.

The equivalence of “ \approx ” in organization **R**, for example, the Council of Ministers of Ukraine (2009), will be affiliated to one of the ministries. Then, the equivalence classes will be all employees of the Ministry of Internal Affairs (MIA), the Ministry of Finance (MFU), the Council of Ministers (RMU), the Ministry of Defense (MDU), etc. Therefore, the Ministry of Internal Affairs, the Ministry of Internal Affairs, the Ministry of Internal Affairs, the Ministry of Education and Science, and others will be the equivalence classes, and the relevant ministers (from the RMU, Julia Volodymyrivna!) Will be elements of the corresponding the factor set.

In general, the factor-set in economic and social studies is interpreted as a table structure of an organization.

4. Matrix incidence of socium

Finally, let \mathfrak{R} be a binary relation on a set **A**, which has **n** elements ($|A| = n$), which are denoted by numbers — **1, 2, . . . , n** and \mathfrak{R} consist of **m** relations ($|\mathfrak{R}| = m$), which are marked with the figures “with circles” — $1^\circ, 2^\circ, . . . , m^\circ$. A **matrix of incidence** $I_m^n(\mathfrak{R})$ is a matrix of dimension $n \times m$, in which **n** columns and **m** lines. Moreover, if the line i° corresponds to the relation $(x,y) \in \mathfrak{R}$, then the intersection of the **i** line and **j** column is **1** if $x = j$ or $y = j$, and **0** in other cases.

Example 5.

$$\text{Let } \mathfrak{R}_1 = \{(1,2), (1,3), (2,3), (2,2), (3,3)\} \tag{4.1}$$

Figure 2 shows the matrix of incidence $I_5^3(\mathfrak{R}_1)$, which illustrates the relation written in (4.1). Denote:

$$(1,2) = 1^\circ, (1,3) = 2^\circ, (2,3) = 3^\circ, (2,2) = 4^\circ, (3,3) = 5^\circ.$$

	Col. 1	Col. 2	Col. 3
Row. 1°	1	1	0
Row. 2°	1	0	1
Row. 3°	0	1	1
Row. 4°	0	1	0
Row. 5°	0	0	1

Figure 2.

Example 6.

Let

$$\mathfrak{R}_2 = \{(1,1), (1,2), (1,3), (2,1), (2,3), (2,2), (3,1), (3,2), (3,3)\} \quad (4.2)$$

In Figure 3, the matrix of incidence $I_9^3(\mathfrak{R}_2)$ is shown, which illustrates the relation written in (4.2). Denote:

$$(1,1) = 1^\circ, (1,2) = 2^\circ, (1,3) = 3^\circ, (2,1) = 4^\circ, (2,3) = 5^\circ, (2,2) = 6^\circ, (3,1) = 7^\circ, (3,2) = 8^\circ, (3,3) = 9^\circ.$$

	Col. 1	Col. 2	Col. 3
Row. 1°	1	0	0
Row. 2°	1	1	0
Row. 3°	1	0	1
Row. 4°	1	1	0
Row. 5°	0	1	1
Row. 6°	0	1	0
Row. 7°	1	0	1
Row. 8°	0	1	1
Row. 9°	0	0	1

Figure 3.

With the help of matrixes of incidence, it is convenient to represent and compile a list of relatively small binary relations. Such lists will be represented by sparse matrices that are effectively packed with known algorithms and can be stored in computer memory without taking up a lot of memory. Therefore, they are often used to build modern **databases** (or **knowledge bank** in modern terminology) of state and interstate ties, inter-party and large groups of in-party relationships.

In the picture 2 of the famous American artist A. Gray, a matrix of incidence of some binary relation in a spherical form is depicted. Spherical incidence matrices are used to add an image of priority relationships in order of decreasing their importance from the upper (north) pole to the equator (typically only the hemisphere is used). Look at our lawn, a more important relationship is depicted on the heart of the Fire man.

5. Ordered of individual data collection

When building the data base of the team, the data collection is carried out individually, for further processing of the received personal information into information for the banks given to the team.

The theory that we begin to explore in paragraph 5 is dual to the theory of ordering in sociometry, which we developed in our previous articles. She investigates not the ordering between the persona, but the ordering of the decisions of one person (in the linear case) of her choices. As a matter of fact, this is exactly what happens during sociological polls, interrogations, questionnaires, psychiatric tests and other classical methods. We consider the mathematical theory of processing individual data collection. Together with the axiomatics, the theorems, their proofs and mathematical implications.

Consider the mathematical model of the process for ordered of individual data collection.

Let α is an individual (a person). Suppose us consider of variants x, y taken in this order. Concerning the triple $(x, y)_\alpha$, the α persons' decision takes



Picture 2. A. Grey. Coordinates of the Fire.

one of the following three forms:

- 1) he/she prefers the variant x to the variant y
 - 2) he/she prefers the variant y to the variant x
 - 3) he/she neither prefers the variant x to the variant y , nor the variant y to the variant x .
- A binary relation

$$x \succ_{\alpha} y \tag{5.1}$$

which symbolizes the statement that the α persons' prefers the variant x to the variant y . A binary relation $x \succ_{\alpha} y$ may be called a **preference relation** of the α person.

A binary relation

$$x \nabla_{\alpha} y = \{(\neg(x \succ_{\alpha} y)) \wedge (\neg(y \succ_{\alpha} x))\} \tag{5.2}$$

which symbolizes the statement that the α persons' is indifferent between the variant x to the variant y and the variant y to the variant x . A binary relation $x \nabla_{\alpha} y$ may be called an **indifference relation** of the α person.

A binary relation

$$x \Xi_{\alpha} y = \{(\neg(x \succ_{\alpha} y)) \vee (\neg(x \nabla_{\alpha} y))\} \tag{5.3}$$

which symbolizes the statement that the α person prefers the variant x to variant y or is indifferent between the variant x to the variant y and the variant y to the variant x . We may call this binary relation $x \Xi_{\alpha} y$ may be called a **strongly preference relation** of the α person.

For classical sociometry, including socionics, which is rapidly developing in the works of outstanding Ukrainian researchers Alexander Bukalov and Karpenko Olga [13, 14], the following axioms are fulfilled.

Axiom 1 asymmetry of preference.

$$\forall(x,y)\forall\alpha \Rightarrow [\{(x \succ_{\alpha} y) \Rightarrow \neg(y \succ_{\alpha} x)\}] \tag{5.4}$$

For any pair of variants (x,y) and any person α , if $(x \succ_{\alpha} y)$ holds, then $(y \succ_{\alpha} x)$ does not holds.

Axiom 2 areflexivity of preference.

$$\forall(x)\forall\alpha \Rightarrow [\neg(x \succ_{\alpha} x)] \tag{5.5}$$

For any pair variants (x) and any person α , $(x \succ_{\alpha} x)$ does not hold.

Axiom 3 transitivity of preference.

$$\forall(x,y,z)\forall\alpha \Rightarrow [\{(x \succ_{\alpha} y) \wedge (y \succ_{\alpha} z) \Rightarrow \neg(x \succ_{\alpha} z)\}] \tag{5.6}$$

For any triple of variants (x,y,z) and any person α , if $(x \succ_{\alpha} y)$ and $(y \succ_{\alpha} z)$ hold, then $(x \succ_{\alpha} z)$ holds.

Axiom 4 transitivity of indifference.

$$\forall(x,y,z)\forall\alpha \Rightarrow [\{(x \nabla_{\alpha} y) \wedge (y \nabla_{\alpha} z) \Rightarrow (x \nabla_{\alpha} z)\}] \tag{5.7}$$

For any triple of variants (x,y,z) and any person α , if $(x \nabla_{\alpha} y)$ and $(y \nabla_{\alpha} z)$ hold, then $(x \nabla_{\alpha} z)$ holds.

Axiom 5 connectedness of strongly preference.

$$\forall(x,y)\forall\alpha \Rightarrow [\{(x \Xi_{\alpha} y) \vee (y \Xi_{\alpha} z)\}] \tag{5.8}$$

For any pair of variants (x,y) and any person α , if $(x \Xi_{\alpha} y)$ or $(y \Xi_{\alpha} z)$ holds.

Axiom 6 transitivity of strongly preference.

$$\forall(x,y,z)\forall\alpha \Rightarrow [\{(x \Xi_{\alpha} y) \wedge (y \Xi_{\alpha} z) \Rightarrow (x \Xi_{\alpha} z)\}] \tag{5.9}$$

For any triple of variants (x,y,z) and any person α , if $(x \Xi_{\alpha} y)$ and $(y \Xi_{\alpha} z)$ hold, then $(x \Xi_{\alpha} z)$ holds.

To form the theory of axioms, it is necessary to be grouped into non-conflicting groups.

Theorem 3.

We have two equivalent and nonconflict groups of the axioms — Axioms 1 and 3, or Axioms 2 and 3.

Proof will be the object of [2, 8].

Other important theorems of the theory of ordering an in-person choice.

Theorem 4.

For any pair of variants (x,y) and any person α , one and only one of the following holds:

$$(x \succ_{\alpha} y), (y \succ_{\alpha} x), (x \nabla_{\alpha} y). \tag{5.10}$$

See the **proof** in [2, 8].

Theorem 5.

For any of variant (x) and any person α , holds:

$$(x \nabla_{\alpha} x). \tag{5.11}$$

The **proof** is trivial.

And

Theorem 6.

For any pair of variants (x,y) , and any person α , $(x \nabla_{\alpha} y)$ holds $\Leftrightarrow (y \nabla_{\alpha} x)$ holds:

$$\{(x \nabla_{\alpha} y) \approx (y \nabla_{\alpha} x)\}. \tag{5.12}$$

The **proof** is left to the reader and see [2, 8].

Theorem 7.

We have two equivalent and nonconflict groups of the axioms — Axioms 1, 3, and 4.

The help for **proof** will be the object of [2, 8].

Theorem 8.

For any triple of variants (x,y,z) , and any person α , if $(x \nabla_{\alpha} y)$ and $(y \succ_{\alpha} z)$ hold, then $(x \succ_{\alpha} z)$ holds:

$$\{((x \nabla_{\alpha} y) \wedge (y \succ_{\alpha} z)) \Rightarrow (x \succ_{\alpha} z)\}. \tag{5.13}$$

The **proof** is left to the reader and see [2, 8].

Theorem 9.

For any triple of variants (x,y,z) , and any person α , if $(x \succ_{\alpha} y)$ and $(y \nabla_{\alpha} z)$ hold, then $(x \succ_{\alpha} z)$ holds:

$$\{((x \succ_{\alpha} y) \wedge (y \nabla_{\alpha} z)) \Rightarrow (x \succ_{\alpha} z)\}. \tag{5.14}$$

4)

The **proof** is left to the reader and see [2, 8].

6. Conclusion

Our axioms and theorems, p.5, transform the classical sociometry of an individual, and this includes all surveys, tests, questionnaires, interrogations and other classical sociometrics in mathematical science. All scientific conclusions should be the result of a complete list¹ of our axioms,



Picture 3. I. Nevimyj. Axioms of humanity.

¹ A complete list of axioms will be presented in the following articles, see also [8].

the theorems and the lemmas of our mathematical theory. Everything else, for example, counting the number of 'yes' respondents in a poll, is at best a heuristic theory.

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Гритсак-Грёнер В.В., Гритсак-Грёнер Ю.
Классическая математическая социометрия. Часть II.
Математика, поэзия, картина

Статья является прямым продолжением статьи «Классическая математическая социометрия. Часть I» тех же авторов и посвящена аксиоматике общественных отношений. Отдельно выделяются классические аксиомы. Традиционный математический словарь дополняет знания читателей об элементах математических отношений. Теоремы 1 и 2 предлагают 24 сценария возможных социальных индивидуальных отношений. Немонотонная тройная спираль жизни человеческих групп основана на трех цепях волн, заряженных до конечной точки. Теория, которую мы начинаем исследовать в параграфе 5, двойственна теории упорядочения в социометрии, которую мы разработали в наших предыдущих статьях. Она исследует не порядок между людьми, а порядок решений одного человека (в линейном случае) в процессе выбора. На самом деле это именно то, что происходит во время социологических опросов, опросов, анкет, психиатрических тестов и других классических методов. Рассматривается математическая теория обработки индивидуального сбора данных, вместе с аксиоматикой, теоремами, их доказательствами и математическими следствиями.

Ключевые слова: отношение, аксиома, социометрия.