SOCIAL PROCESS MODELING

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CLASSICAL MATHEMATICAL SOCIOMETRY. PART I

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The main purpose of the article is to study mathematical properties sociological relations with a relatively small number of people. For the autonomy of ours article we provide a sufficient mathematical glossary. According to the Complexity of Control Principle (CSP) [1], the complexity of the managed object is not less than the complexity of the control object. We propose min classical methods management of society. The authors are infinitely grateful for the comments Professor Timofeyev-Resovsky N.V.

Key words: set, union, socium.

0. Mathematical glossary

Sets and all

Lo in the orient when the gracious light
Lifts up his burning head, each under eye
Doth homage to his new-appearing sight,
Serving with looks his sacred majesty;

William Sakespeare

Theory sets first arose as a language for describing geometrical properties of combinatorial objects. The emergence of a new language is always an important event in the development of applied mathematics.

Although it is customary in mathematics to treat the words "**set**" and "**element**" as undefined terms. By native or intuitive definition ("set paradise, see Picture 1"), we think of a **set** as something made up by all the objects that satisfy some given condition. **Fréchet** introduced sets in end XIX country. For example, the following are sets:

(1) all simple numbers — **P**,
(2) all quarter notes "do" in Petro Tschaikowskij concerto N1 of a clavier — $_4\mathbf{C}^1$,
(3) all fairies strictly contained in Klein bottle — $\mathbf{F}^b$,
(4) all holies in Bosch's picture, see picture 1.

Such well-defined collections of distinguishable objects serve as intuitive interpretations of the word "**set**", while the objects themselves are examples of the "**elements**" of a set. Sets may be **finite** or **infinite**. We describe a **finite set** as having **i** elements, where **i** is a nonnegative integer. Neither of these sets consists of a finite number of elements; hence they are called **infinite sets**. By definition the **empty set** with no elements; it is generally denoted by ∅.

We usually denote sets by capital letters (**A**, **B**, **PH**, $_4\mathbf{C}^1$, **H**) and the elements of a set by lowercase letters (**a**, **α**, $\boldsymbol{\beta}_i$).

We can represent a set **A** and a set **B** by closed curves within the rectangle as shown in Figure 1; such diagrams are often called **Wienn diagrams**.

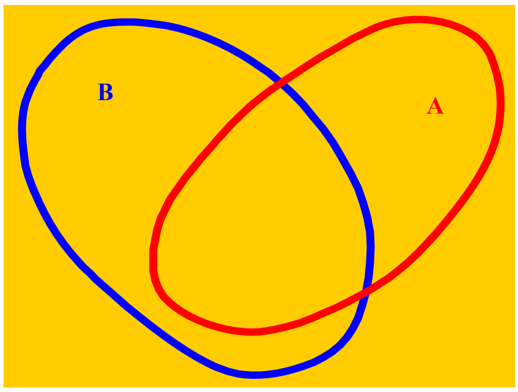

Figure 1. Wienn diagram.

Definition 0.1. If **A** is a set, we write **a∈A** (equivalent **A∋a**) to indicate that **a** *is an element of* **A**.

Let **A** and **B** be sets. Set **A** is a **subset** of a set **B** if every element of **A** is also an element of **B** and write **A ⊆ B** or also **B ⊇ A**. Thus, **A** is a **proper subset** of a set **B** if **A ⊆ B**, **b∈B** and **b∉A,** we write **A⊂B** or **B ⊃A**.

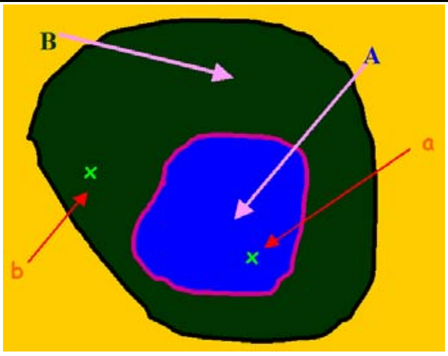

Figure 2.

In the **contrary** cases we write **a∉A**, **A ⊆ B**, **A ⊈B**.

Example 0.1. (See Wienn diagram in Figure 1). If **B** represents the set of all living wolves in the world and **A** represents the set grey wolves, then we can say

A ⊆ B.

If **B** represents the set of all living wolves in the world and **A** represents the blue wolves, then we can say **A ⊂ B**. The element **b∈B** assign the wolf. The red wolf corresponds to the element **a∈A**.

There are several ways that a set can be defined. First, a defining statement can be used: "**A** is the set of the holies in picture 1" defines a set **A**. Second, the same set can be defined by listing the elements within braces; for example,

$$\mathbf{A= \{h_1, \ldots , h_n\}}. \qquad \mathbf{(0.1)}$$

Third, using a colon to represent the expression "such that", we can use the builder notation; for example,

$$\mathbf{A} = \{\mathbf{h_i : h_i} \text{ is one of the holies in pictures 1}\} \qquad \mathbf{(0.2)}$$

or

$$\mathbf{A}= \{\mathbf{h_i : h_i}\in H^1\}, \qquad \mathbf{(0.3)}$$

where is the set all holies in picture 1.

Definition 0.2. Let **A** and **B** be sets.

Two sets **A** and **B** are said to be **equal** if their elements are same and will be designed by **A** = **B**.

The set of elements that belong to both **A** and **B** is called the **intersection** of **A** and **B** and is denoted by **A⋂B.**

The set of elements that belong to **A** or **B** is called the **union** of **A** and **B** and is denoted by **A⋃B**.

From two sets **A**, **B** we can form another set, the **difference**, written **A\B**, which consists of all elements in **A** that are not in **B**.

Let **A ⊆ B**. The set of all elements that belong to **B** but do not belong to **A** is called the **complement** of **A** in **B** and is denoted by $\vartheta_{\mathbf{B}}\mathbf{A}$.

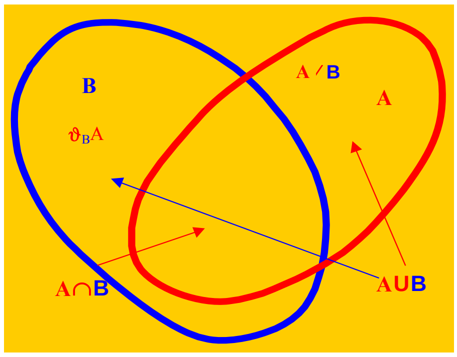

Figure 3.

Example 0.2. We have the Wienn diagram representation **A⋃B**, **A⋂B**, **A\B**, $\vartheta_{\mathbf{B}}\mathbf{A}$ as shown by the colored region in Figure 3.

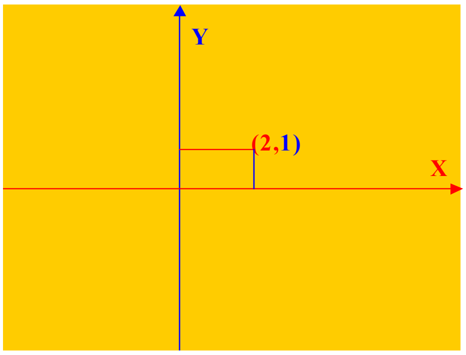

Figure 4. Cartesian plane (R×R).

Definitions 0.1.2 can be extended to define the equal, intersection, union, difference and complement of three or more sets and are denoted

A = B = C = ... ,

A ⋂ B ⋂ C ⋂... or $\bigcap_i A_i$, **i∈I,**

A ⋃ B ⋃ C ⋃... or $\bigcup_i A_i$ $\bigcup_i A_i$, **i∈I,**

A \ B \ C \ . . . ,

$\vartheta_{\mathbf{B}}\mathbf{A}\ (\vartheta_{\mathbf{C}}\mathbf{B}\ (\vartheta_{\mathbf{D}}\mathbf{C}\ \ldots))$

and mixed.

Definitions 0.3. From any two elements **a** and **b** we can form the sequence **(a,b)**; it is called an **ordered pair**, and of course is different from **(b,a)**, unless **a =b**.

Let **A** and **B** be any sets. We denote by **A×B** the set of all ordered pairs **(a,b)** with **a∈A** and **b∈B**. The set **A×B** is called the **Cartesian product** of **A** and **B**. More generally, from n sets $\mathbf{A_1}, \ldots,$ $\mathbf{A_n}$ we can form the **Cartesian product**

$$\mathbf{A_1} \times \ldots \times \mathbf{A_n} \text{ or } \prod_{1}^{n} A_i \qquad \mathbf{(0.4)}$$

Example 0.3. Let **R** be the set of all real numbers. The **Cartesian** product **R×R** is the set of all ordered pairs of real numbers. The elements **(a,b)** of **R×R** are used to identify points in the **Cartesian** plane. For example, the **Cartesian** plane and element **(2.1)** graphed in Figure 4.

1. Relations and some

The theory of relations, how often it is called the theory of lattices, in accordance with the name of its founder, the prominent American mathematician G. Birkhof, is one of the most fundamental mathematical sciences, which has numerous applications in all sciences.

A **society S** consists of individual persons

$$\mathbf{P = \{p_1, p_2, \ldots, p_n\}},$$

relations between persons of society

$$\mathbf{R = \{r1, r2, \ldots, r_{n\times n}\}}.$$

And society's decision is composed of its persons' decisions

$$\mathbf{D(\{p_1, p_2, \ldots, p_n\})} \text{ or } \mathbf{D(P)}.$$

Management or **control C(S)** over society **S** is a function

$$\mathbf{F: P \cup R \rightarrow \Theta}$$

where Θ there are all the important decisions of society **S**.

On the other hand, it is a pairwise and more array relationship between members of society and are an integral part of social, economic and state relations.

Actually, in their totality and create the essence of what is called the Society, Business and the State. Therefore, it is obvious that the theory of relations should be the most popular mathematical discipline in humanitarian and economic applications. The guilty reason is the bad mathematical education of our humanitarians, which we, practically first, provide the elements of this extremely necessary mathematical theory.

Figure 5. Hieronymus Bosch, The Garden of Earthly Delights.

The most popular binary relation is the row-by-line relationship for numeric networks, which is denoted by "≥". For example: $7 \geq 3$, $\pi \geq 3$, $50\% \geq 40\%$, and the like. But in reality, see example 0.2.2, the notion of binary relation is much more general.

Definition 1.1. The binary relation $\mathfrak{R}$ for set A is the subtree of the Cartesian product $\mathfrak{R} \subseteq A \times A$, whose elements will be pairs (a_1, a_2), $a_1, a_2 \in A$. If $(a_1, a_2) \in \mathfrak{R}$, then we will denote how

$$\mathbf{a_1 \succ a_2} \qquad \mathbf{(1.1)}$$

Example 1.1. For example, for a set **A** = {1,2,3} of the three elements in binary relations will be $\mathfrak{R}_1$ and $\mathfrak{R}_2$

$$\mathfrak{R}_1 = \{(1,2), (1,3), (2,3), (2,2), (3,3)\} \qquad \mathbf{(1.2)}$$

$$\mathfrak{R}_2 = \{(1,1), (1,2), (1,3), (2,1), (2,3), (2,2), (3,1), (3,2), (3,3)\} \qquad \mathbf{(1.3)}$$

Example 1.2. In the process of managing the state and large social, party and economic associations, there are clear problems at every step definition and further research to make the best solution to the following problems, to which we will often return in our work.

(a) Representation of the set $\mathbf{D_p}$ of all possible (or available) solutions of the problem **p** to the pair $\mathbf{d_i \succ d_j}$, where $\mathbf{d_i}$ is a better solution than the solution $\mathbf{d_j}$ of our problem. Note that the binary relation "**≻**" can not be replaced here by the relation of order "≥". After all, there is a case where the solution $\mathbf{d_1}$ is better than $\mathbf{d_2}$, $\mathbf{d_2}$ is better than $\mathbf{d_3}$, and $\mathbf{d_1}$ in general $\mathbf{d_3}$ is not comparable to each other, in other words

$$(\mathbf{d_1},\mathbf{d_3}) \notin \succ \qquad \textbf{(1.4)}$$

omen frames, is not defined at all by "≥".And this will be the case in all the following examples. The reader will find counterexamples on his own.

(b) Selection from set **F** of all possible political decisions such pairs

$$(\mathbf{p_i},\mathbf{p_j}),$$

where $\mathbf{p_i}$ the previous decision, and $\mathbf{p_j}$ the next so that the result was a program, or the intended purpose of the government or other leadership.

(c) Product quality control, or enforcement of decisions. In both cases, the pair of relations

$$(\mathbf{v},\mathbf{r_i}),$$

where **v** is the reference standard sample (the control number of the plan), and $\mathbf{r_i}$ product that has come to the control (the state of execution of the decision at the control moment) is formed.

(d) Comparative calculations for the use of investments, or vice versa, to reduce the financing of certain branches of production or social processes.

(e) Comparative characteristics for continuing or stopping an economic or social experiment.

	1	2	3
1	0	1	1
2	0	1	1
3	1	0	1

	1	2	3
1	1	1	1
2	1	1	1
3	1	1	1

Figure 6.

At first glance (e) is not difficult to solve as it is only a partial case of example (d). But here, for example, are the problems of closure, or the continuation of the existence of free economic zones in Ukraine, even the problem of the continuation or termination of the construction of communism and communist society. In this example, we are confronted with the concept of the **degree of responsibility** of a solution, which we will also study in our book. Let's say in advance that class (e) problems usually have a much greater degree of responsibility than (d), so they are no less complex.

In the case where **A** is a finite set whose elements are successively listed with integers, the binary relations $\mathfrak{R}$ are conveniently represented in the form of **incidence matrices** of the size $|\mathbf{A}| \times |\mathbf{A}|$, in which rows and columns are designated by elements **A**, and in a cell with coordinates

$$[\mathbf{r_1},\mathbf{r_2}],\ 1 \le \mathbf{r_1},\ \mathbf{r_2} \le |\mathbf{A}|$$

is "**1**" if the pair $(\mathbf{r_1},\mathbf{r_2})$ is an element $\mathfrak{R}$ and "**0**", otherwise. For example, in Fig. 6, the matrices of the incident binary relation (1.2) and (1.3) are depicted.

Significantly less often than binary relations, in studies there are **n-relations**. Most likely, this is related to the primitive "plane" of the onset of thinking. In rare cases of non-binary relations, we will always be separately paying attention to readers.

Definition 1.2. The n-relation $\mathfrak{R}^{n}$ for set A is the subset of the Cartesian n-product

$$\mathfrak{R}^{n} \subseteq \mathbf{A} \times \ldots \times \mathbf{A}$$

elements of which will be ordered **n**-type

$$(\mathbf{a_1}, \ldots, \mathbf{a_n}),\ \mathbf{a_1}, \ldots, \mathbf{a_n} \in \mathbf{A}.$$

If $(\mathbf{a_1}, \ldots, \mathbf{a_n}) \in \mathfrak{R}^{n}$, then $(\mathbf{a_1}, \ldots, \mathbf{a_n})$ we will denote as

$$(\mathbf{a_1}, \ldots, \mathbf{a_n})^{\mathfrak{R}}. \qquad \textbf{(1.5)}$$

Example 1.3. The problem of finding a suitable groom or bride is a classic question of constructing optimal binary relations. But, when this problem comes with more responsibility and is selected, in addition to the classic pair **(groom, bride)** as well as other relatives, we at least get the problem of finding optimal 8-relations: **(fat, mother-in-law, groom, bride, father-in-law, mother-in-law).**

Undoubtedly, such a problem is much more difficult than the already complex problem of the class.

2. Binary relations

A personal decision has been defined as a personal preference ordering of all conceivable alternatives. Wes hall start just objects of decision making, i.e. a fixed set **A** with elements $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\boldsymbol{\gamma}$, … . We assume there is a basic set of alternatives which could conceivably be presented to every person as well as to the society. This set of all conceivable alternatives may be called a ***possibility*** and denoted by $\mathbb{P}$. For any given possibility, a decision my be visualized as a relation among alternatives in it. We shall show in the following that a preference is a ***binary relation*** between two alternatives.

Definition 2.1. A ***binary relation*** (on **A**) is a subset $\succcurlyeq$ of **A** × **A**. Frequently we write "$\boldsymbol{\alpha} \succcurlyeq \boldsymbol{\beta}$" $\overset{iff}{\Leftrightarrow}$ $(\boldsymbol{\alpha},\boldsymbol{\beta}) \in \succcurlyeq$ when $\boldsymbol{\alpha},\boldsymbol{\beta} \in \mathbf{A}$. If $\boldsymbol{\alpha} \succcurlyeq \boldsymbol{\beta}$, then we shall say $\boldsymbol{\alpha}$ is ***preferred*** to $\boldsymbol{\beta}$.

Example 2.1. Let us consider a pair of alternatives (**Bob**, **Sam**). The i-th personal's decision takes one of the following three forms: she(he) prefers alternative **Bob** to alternative **Sam**, she(he) prefers **Sam** to **Bob**, or she(he). We may introduce a binary relation **Bob** $\succcurlyeq$ **Sam** which mean the statement that the i-th persona prefers alternative **Bob** to alternative **Sam**.

Definition 2.2. A binary relation on **A** is called

1) **reflexive** if $\alpha \succcurlyeq \alpha$,
2) **irreflexive** if $\alpha \not\succcurlyeq \alpha$,
3) **complete** if $\alpha \succcurlyeq \beta$ or $\beta \succcurlyeq \beta$ holds true for any $(\alpha,\beta) \in A \times A$,
4) **transitive** if $\alpha \succcurlyeq \beta$, $\beta \succcurlyeq \gamma$ implies $\alpha \succcurlyeq \gamma$ $(\alpha, \beta, \gamma \in A)$,
5) **indifferentic** if $\alpha \not\succcurlyeq \beta$ and $\beta \not\succcurlyeq \alpha$,
6) **symmetric** if $\alpha \succcurlyeq \beta$ implies $\beta \succcurlyeq \alpha$,
7) **antisymmetric** if $\alpha \succcurlyeq \beta$, $\beta \succcurlyeq \gamma$ implies $\alpha=\beta$,
8) **asymmetric** if $\alpha \succcurlyeq \beta$ implies that $\beta \succcurlyeq \alpha$ does not hold true.

Example 2.2. Let us consider a pair of alternatives (**Bob**, **Sam**). The *i*-th personal's decision we denoted **Bob** $\succcurlyeq_i$ **Sam**. Then the binary relation if the following conditions hold:

(i) irreflexivity;
(ii) transitivity;
(iii) antisymmetry.

Figure 7. S. Dali. Bride's Choice.

Definition 2.3. A binary relation $\succcurlyeq$ on **A** is called a **preference** if $\succcurlyeq$ reflexive, transitive, and complete.

Example 2.3. Let a peoples socium S consists of individual persons $\mathbf{m_i}$, $\mathbf{i} \in \mathbf{I}$. Suppose mean $\mathbf{m_i} \succcurlyeq_f \mathbf{m_j}$ if $\mathbf{m_i}$ familar of $\mathbf{m_j}$. Then the binary relation is preference.

Definition 4. Let a binary relation $\succcurlyeq$ on **A**. Then

a. $\succcurlyeq^\star = \{(\boldsymbol{\alpha},\boldsymbol{\beta}): (\boldsymbol{\beta},\boldsymbol{\alpha}) \in \succcurlyeq\}$ is called a dual relation of $\succcurlyeq$,
b. $\succ = \succcurlyeq \cap \not\succcurlyeq^\star$,
c. $\approx = \succcurlyeq \cap \not\succcurlyeq^\star$.

3. A collective solution is generated by an individual

A **socium S** consist of individual persons and socium's solution is composed of its individuum's solution [1, 4, 5].

A **peoples socium S** consists of individual persons. Formally, we may express this as

$$\mathbf{S} = \{\mathbf{m_i} | \mathbf{i} \in \mathbf{I}\} \qquad \textbf{(3.1)}$$

where $\mathbf{m_i}$ are persones for the decisions, **I** is the set of indetification numbers for persones. For example, **I** is the set of numbers of its pass.

A **socium's future solution R** express as

$$\mathbf{R} = \{\mathbf{r_j} | \mathbf{j} \in \mathbf{J}\} \subseteq P(\mathbf{S})^{P(\mathbf{S})} \qquad \textbf{(3.2)}$$

where $P(\mathbf{S}) = 2^{\mathbf{S}}$.

Any socium has its own rule for making solutions. When a combination of individual solutions is given, A sociums solution is reached according to that rule.

The most simple of a social relation rule is voting. By casting a ballot, any individual expresses his solution on the issue in question, say, an election where a Regional candidate and a BJUT candidate are contesting a seat in President's post. Ukrainian's socium adopts the decision supported by the majority of voters; thus if the Regional candidate are obtains more votes, he is the "Ukraine choice".

The mathematician would say that a socinm solution is a **functor F** of individual solutions, in the sense that a combination of individual solutions determines a socium solution. More strictly, denote by **F** the functor that takes each individuum category **S** (see (2.1)) to future's socium category **R**.

$$\mathbf{F : S \to R.} \qquad \mathbf{(3.3)}$$

The paper proposes an algorithmic-structural theory [2] for the study of the functor **F** and the category **R** in (2.3).

4. A societ's decision is composed of its persons' decision

Commentary. We note for those who are well versed in modern mathematics that **R(S*)** in specific cases is a complex logical predicate of universal algebra. And we will learn, by the end of the first volume, to construct these predicates. When the well-known **quier of the social decision K** is a natural-numeric vector of the length **μ(S*)** of the result of a social solution:

$$\mathbf{K} = (\rho_1, \rho_2, \ldots, \rho_e), \qquad \mathbf{(4.1)}$$

where **e** = **μ(S*)**, $\rho_i \in$ **A** is the **result of the solutionof one question**, or an **alternative** to the i-th member of the subjectivity, which has the right to vote. If, **μ(A)=1**, then the social solution is called **non-alternative**. In the classical case, **A** is a Boolean, so in the finite case there are **2, 4, 8, ..., 2n** elements, **n** $\in \mathbb{Z}^+$. Obviously, these are variants:

A (2) — «**yes**» or «**no**» from two alternatives;

A (4) — «**yes**», «**no**», «**all the same**», and «**did not vote**» from four alternatives.

In the vast majority of cases, there are real choices, only two and four alternative results, although sometimes more and even infinitely elemental.

Finally, the choice of society is determined by the algorithm

$$\mathbf{Alg(K)}, \qquad \mathbf{(4.2)}$$

the input data of which is the whim of the public decision **K**, and the algorithm (4.3) is determined by the decision-making rule, which is adopted in a society adopted in a society to which peoples socium S. belongs.

Figure 7. Grosh. Political Socium.

5. Main rules of democratic social choice

Once a pattern of individual decision or control is formed, the rule of majority voting yelds a social decision. As the pattern changes, the social decision also varies.

Any change in social decision can occur only through changes in personal decision. Thus, we can express any social decision rule.

The simple rule of 'decision by majority' can be made complicated in several ways:

VETO RULE. Granting veto power to some participants[1].

TWO RULE. Requiring a majority by two different measures[2].

[1] (e.g. the permanent members of the UN Security Council or the President of the Ukraine, or the Manager of the SKY in Charkiv), possibly subject to 'override' by a sufficiently large majority of another body (e.g. the Central Rada, or Board of Directors for SKY).

[2] (e.g. in a republicanian system, and a majority of provinces or a consumer interest and safety interest).

WHEIGHT RULE. Giving different weight to different voters[3].
FORCING RULE. Forcing voters to vote in 'blocs'[4].

6. Conclusions

We can express any sociometric decision rule.

Sometimes, as in the case of political taboos, sociometric decision may be traditionally fixed, regardless of change in personal decisions. However, a traditionally democratic fixed sociometric decision is still a functor of personal decisions (in the sence that functor takes on function of a constant value).

In some society (psevdodemocratic), particular persons may be so powerful that his decision is always adopted by the society. In other words, the person is controler or dictator.

But this controler (dictatorial) rules of sociometrical decision is simply a special class of sociometric decision functor. Whether a society is traditional, controler, dictatorial or democratic, we shall investigates later in next articles.

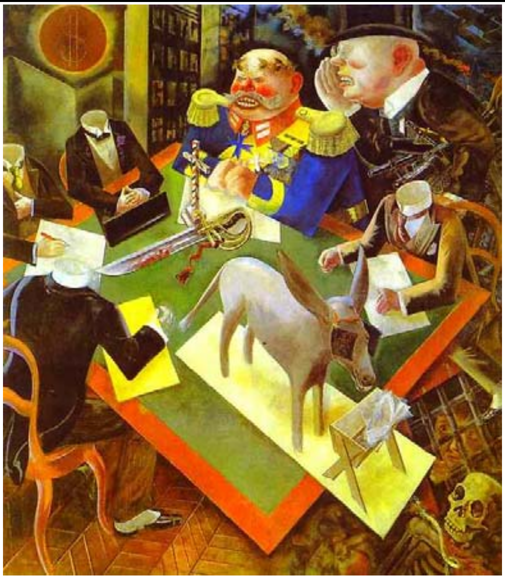
Figure 8. Grosh. Controler.

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Классическая математическая социометрия. Часть I

Основной целью статьи является изучение математических свойств социологических отношений относительно небольшого числа людей. Для автономии нашей статьи мы предоставляем достаточный математический глоссарий. Согласно сложности принципа управления (CSP) [1], сложность управляемого объекта не меньше сложности управляющего объекта . Мы предлагаем минимальные классические методы управления обществом. Авторы бесконечно благодарны за комментарии профессора Тимофеева-Ресовского Н.В.

Ключевые слова: множество, объединение, социум.

[3] (e.g. different aktivistes in a publically traded corporation, or different states in the EU).

[4] (e.g. political parties, finanical concerns).