SYNERGETICS AND THEORY OF CHAOS

УДК 167.7, 517

Gritsak von Groener V. V., Gritsak-Groener J.

STRATEGICALLY CONTROL OF CHAOS AND INVERSE PROBLEMS

HRIT Laboratory, International EBS Laboratory (Germany, Austria, Switzerland); University of Georgia (USA)
e-mail: v_hrit1000000@yahoo. com

We introduced the notions control of chaotics, i.e. control the finite chaos structure. Further we introduce the notion control strategy. In section 2 review some of the standard facts on control for chaotics. In section 3 have complied some basic facts of chaos flows control with penalty function. Section 4 is devoted to the study of control strategy against external controller and antiterrorist control strategy. Let us the groundset $\mathbf{A} \subset \mathbb{Z}\times\mathbb{Z}$ to the case under discussion in section 5. In section 6 we gave the direct algorithm of single-center infection on $\mathbb{Z}\times\mathbb{Z}$ with the ramified boundary of the ground-set $\mathbf{A} \subset \mathbb{Z}\times\mathbb{Z}$. Finally in section 7 we gave inverse algorithm for computational disaster advances(**DA**)of single-center infection on $\mathbb{Z}\times\mathbb{Z}$ with the ramified boundary of the ground-set $\mathbf{A} \subset \mathbb{Z}\times\mathbb{Z}$. Also we designed and developed a set of algorithms for construction of the arbitrary and concrete chaotic set that can efficiently be used in evaluations of the propagations autooscillatory geotectonic waves.

Key words: chaos, chaotic, algorithm.

Nichts war noch vollendet, eh ich es erschaut,
ein jedes Werden stand still.
Meine Blicke sind reif, und wie eine Braut
kommt jedem das Ding, das er will.

Nichts ist mir zu klein und ich lieb es trotzdem
und mal es auf Goldgrund und groß,
und halte es hoch, und ich weiß nicht wem
löst es die Seele los...[1]

Rilke

1. Introduction

Let the set B be the disaster advances. We will consider the general direct algorithm for viral extension and other disaster advances (**DA**) is given by the closer of the set $\mathbf{B} \subseteq \mathbf{A}$ for the chaotic a chaotic $\mathbf{H} = (\mathbf{A}, \mathbf{\Omega})$, $\mathbf{\Omega} \subseteq 2^{\mathbf{A}}$. Let **t** is an iteration number of **DA**. Hence **B** is the disaster zone. The **DA** takes the extension **B** to $\backsim$ **(B)**

$$\mathbf{B} \subseteq \mathbf{C_1} \subseteq \ldots \subseteq \mathbf{C_i} \subseteq \ldots \subseteq \mathbf{C_t} = \backsim(\mathbf{B}) \subseteq \mathbf{A}, \qquad (1)$$

where $\mathbf{C_i}$ is the part closer of **B**. Cycles χ_k are elements of $\mathbf{\Omega}$. Let χ_t be given by $\mathbf{C_{t+1}} \supseteq \chi_t \not\subset \mathbf{C_{t+1}}$. Then χ_t is interpret of the disaster source. The inverse algorithm for computational disaster advances (**IDA**) is given by coordinates of infection sources $\{\chi_t\}$, $\mathbf{t} = [1, n]$, $\mathbf{n}=\mu(\mathbf{\Omega})$.

2. Glossary

For convenience of the reader we repeat the relevant material from [5].

Let **A**, **B** be the sets. A $\mathscr{F}$**: A** ━¤━▶ **B** is the **multimap(mm)** $\mathscr{F}$**: A** $\longrightarrow$ $\mathbf{2^B}$.

---

1 Ничто - вне прозрений моих - не в счет:
застыв, каменеет путь.
Лишь к зрелому зрению притечет
вещей вожделенная суть.

Ничто мне - ни что. Но любя его, я
на фоне пишу золотом:
чью душу восхитит? - и тьма ли Твоя? -
огромный неведомый дом...
перевод А. Прокопьева

The pair $\mathbf{G} = (\mathscr{F}, \mathbf{A})$ is called a **control graph G** of **mm** $\mathscr{F}$. The elements of the set **A** are a ***nodes*** of **G**. The pairs $\mathbf{u} = (\alpha, \mathscr{F}(\alpha))$ αare called an **arrows** of **G**, where the **α** is a **tail** of **u** and $\mathscr{F}(\alpha)$ is a **spike** of **u**. $\mathbb{N}(n) \overset{def}{=} \{k\in\mathbb{N}: k \le n\}$. $\mathfrak{R} = (A_i : i\in\mathbb{N}(n))$ is called an **indexed family**. An indexed family $\mathfrak{R}$ is called a **personal family** if $A_i \ne A_j$ when $i \ne j$. By definition put $A_i^c = A \setminus A_i$. A family sets $S^c = \{A_i^c : i\in\mathbb{N}(n)\}$ is called a **complement of** the $S = \{A_i : i\in\mathbb{N}(n)\}$.

Suppose

$$\mathscr{H} = (A : C_i, i\in\mathbb{N}(n)) \qquad (2)$$

is an personal family such that

(1) $\mu(C_i) \ne \varnothing$,

(2) if $C_i \subseteq C_j \Rightarrow C_i = C_j$ when $i \ne j$.

We call $\mathscr{H}$ a ***chaotic*** (***chaos***) on the set **A**. A chaotic **M** on the set **A** is the chaotic of ***circuits*** of a ***matroid***

⇒ ≠ℭℵ≻∩∪ ℕ

$$\mathbf{M} = (A, \mathfrak{C} = \{Z_i\}: C_i, i\in\mathbb{N}(n)) \qquad (3)$$

on the set **A** if $\varnothing \notin \mathfrak{C}$ and $\mathfrak{C}$ satisfies the ***elimination axiom*** :

(**ax**) whenever $Z^1 \ne Z^2\in\mathfrak{C}$ and $A\ni\alpha\in Z^1 \cap Z^2$, there is a $Z^0\in\mathfrak{C}$ with $Z^0 \subseteq Z^1 \cup Z^2\setminus\{\alpha\}$.

A binary relation ≽ on **A** is called a ***preference*** if ≽ reflexive, transitive, and complete. Let ≻ be a strongly binary relation on **A**. Then an ***acute hull*** ⋙ of $\overset{iff}{\Leftrightarrow}$ there exists a sequence $\alpha = \alpha_0, \ldots, \alpha_n = \beta$ such that $\alpha_I \succ \alpha_{i+1}$ $(i\in\mathbb{N}(n-1))$. For every fixed $\alpha^*\in A$ let $\mathfrak{I}(\succcurlyeq, \alpha^*) = \{\alpha\in A : \alpha^* \succcurlyeq \alpha\}$. Similarly, $\mathfrak{I}(\succ, \alpha^*) = \{\alpha\in A : \alpha^* \succ \alpha\}$.

Let **U** is a finite set. A ***digraph*** D is a pair is a pair $D = (U, \succ)$. A ***ditree*** $T^{\succ}$ is a digraph $(U, \succ)$ such that there exist an element $\alpha^o\in U$ (to be called a ***root*** of the digraph) having the following properties:

Figure 1. Dmytry Pollack. Disaster Zone

a. $\alpha \ggg \alpha^\circ$ $(\alpha\in U)$,

b. $\mathfrak{I}(\succ, \alpha^\circ) = \varnothing$,

c. $\mu(\mathfrak{I}(\succ, \alpha)) = 1$ $(\alpha \ne \alpha^\circ)$.

The elements of the set **U** are a ***vertex*** of $T^{\succ}$. The pairs $\mathbf{u} = (\alpha, \beta)$ are called an ***arrows*** of $T^{\succ}$ if $\mathfrak{I}(\succ, \alpha) = \beta$.

$\mathbb{Z}\times\mathbb{Z}$ is called **square lattice** over $\mathbb{Z}$, where $\mathbb{Z}$ a ring of integer numbers. The ground-set $A \subseteq \mathbb{Z}\times\mathbb{Z}$. Now consider a graph $\Gamma = (V, E)$, where the vertex-set $V = A$ and edge-set $E = \{e\in B, e \in\{\forall(i,j) \in V: (i-1,j), (i,j-1), (i+1,j), (i,j+1)\}\}$. Before consider the **inverse function** on a graph **Γ**, which can be written in the form

$$\aleph(\Gamma) = \sum_{\pi} \prod_{(i,j)\in E} \theta_{\pi_i \pi_j} \qquad (4)$$

where $\pi_i\pi_j$ is either **1** or **–1**, $\boldsymbol{\theta} = \mathbf{e}^{\mathbf{I}}$, here **I** is a number of iterations.

Let W be the infection's network on square lattice $\mathbf{Q}^2$ with ground-set $\mathbf{A} \subseteq \mathbb{Z}\times\mathbb{Z}$. Suppose that there is a supply of **disarder fluid** (**df**) at the origin and that each edge of $\mathbf{Q}^2$ allows fluid to pass along it with probability **p**, independently for each edge. Let $\mathbf{P_i(p)}$ is the probability that **vf** spreads to at least **i** vertices. Thus

$$\mathbf{P}^{\mathbf{W}} = \lim_{\mathbf{i}\to\infty} \mathbf{P}_{\mathbf{i}}(\mathbf{p})$$

is called a critical probability of **W**.

Proposition 1. [See 5]. The critical probability that **vf** spreads to at least vertices on the square lattice **W** is between **0.51** and **0.68**.

We shall say that the chaotic (2) is a **controlled chaos** $\mathscr{H}$, where [**i**]$\in$ **ℕ(n)** is a **controller**, the index family ($\mathbf{C_i}$, **i**$\in$ℕ(**n**)) are a **territory of the controller** [**i**], and **Z = {[1]**, …, **[n]}** is a **control-brigade** (or **brigade**). The elements of set **A** are a **position of control** for the chaotic

$$\mathbb{H} = (\mathbf{A} : \mathbf{C_i}, \mathbf{i} \in \mathbb{N}(\mathbf{n})),$$

A is a **position-set**. Suppose the pair ($\mathbf{Z}^1$, $\mathbf{Z}^2$) is partition **Z** when $\mathbf{Z}^1$ are an **active controllers**, and $\mathbf{Z}^2$ are an **passive controllers**.

We shall say that for the chaotic ℍ there exists a **control** if the following conditions hold:

(a) we have a multimap

$$\mathbf{\Omega} : \mathrm{A} \multimap \mathrm{A}, \qquad (5)$$

then this is called a **law of the control**;

(b) for any **[i]** there exists a preferences $\succcurlyeq_i$, then this $\succcurlyeq_i$ is called a **preference** of controller **[i]**.

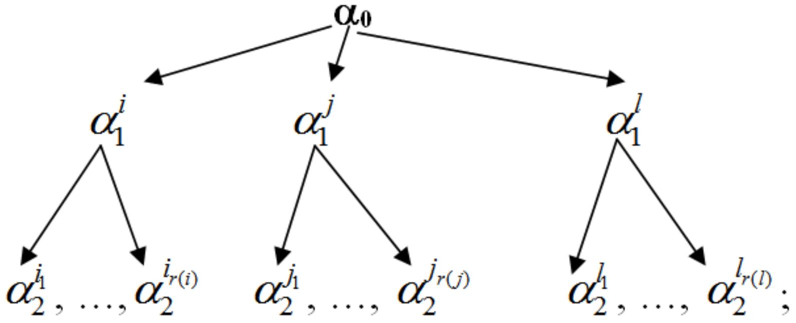
Let $C_0 \overset{\text{def}}{=} (\alpha\text{: } \mathbf{\Omega}(\alpha)=\varnothing)$ and using a transformation of **Ω** we get $\mathbf{\Omega}(C_0) \cap C_k = \emptyset$, where $k\in\mathbb{N}(n)$.

Suppose $\boldsymbol{\alpha_0}\in\mathbf{A}$ be a ***beginning*** element of position. We shall say that a **brigade Z = {[1]**, . . ., **[n]} experts control** over the chaotic ℍ if the following steps hold:

(1) let **([i], …,[j], …,[l])** ≠ ∅ (**1≤ i ≤ … ≤j≤…≤ l≤n**) is maximum allowable of controller number such that $\mathbf{\Omega(\alpha_0)} \cap \mathbf{C_t} \neq \varnothing$, where **t∈(i, …, j, …, l)** whence the controller **[t]** ***choose*** element of position $\alpha_1^t \in \mathbf{\Omega(\alpha_0)}$, control is continue and we have the controllable positions $\boldsymbol{\alpha_0}$, α_1^i, …, α_1^j, …, α_1^l;

(2) if ([i], …,[j], …,[l]) = ∅ the control is *finished*;

(3) let t∈(i, …, j, …, l), if ([i_1], …, [$i_{r(i)}$], …,[j_1], …, [$j_{r(j)}$], …,[l_1], …,[$l_{r(l)}$]) ≠ ∅ is maximum allowable of controller number such that $\mathbf{\Omega}(\alpha_1^t) \cap \mathbf{C}_{t_1} \neq \varnothing$, where $\mathbf{t_1\in(i_1, …, i_{r(i)}, …,j_1, …, j_{r(j)}, …,l_1, …,l_{r(l)})}$ whence the controller **[t]** ***choose*** element of position $\alpha_1^{t_1} \in \mathbf{\Omega}(\alpha_1^t)$,control is continue and we have the controllable positions



(4) if ([i_1], …, [$i_{r(i)}$], …,[j_1], …, [$j_{r(j)}$], …,[l_1], …,[$l_{r(l)}$]) = ∅ the control is *finished*;

(5) and so on, as so long the controller induce the nonempty positions in the ditree

$$\mathbf{T}^{\succ} = (\mathbf{U}, \succ) \qquad (6).$$

A ***depth*** of ***control*** is called a lenght of maximal depath in $\mathbf{T}^{\succ}$.

A preference $\succcurlyeq_i$ of controller **[i]** it is possibility represent such that a numerical bounded function $\mathbf{f_i(x)}: \mathbf{x} \longrightarrow \mathbb{R}$ as follows: $\alpha \succcurlyeq_i \beta \Leftrightarrow \mathbf{f_i(\alpha)} \geq \mathbf{f_i(\beta)}$. Then we shall say that a brigade **Z = {[1], . . ., [n]} experts control with penalty function $\mathbf{f_i(x)}$** over the chaotic $\mathbb{H}$. If a controller **[i]** is the active, it is customary were more preferable to position of control with respect to $\succcurlyeq_i$, $\mathbf{f_i^+(C_i)} = \sup_{x\in C_i} \mathbf{f_i(x)}$ will be written in terminology of penalty function. If a controller **[i]** is the passive, it is not customary were less preferable to position of control with respect to $\succcurlyeq_i$, $\mathbf{f_i^-(C_i)} = \inf_{x\in C_i} \mathbf{f_i(x)}$ will be written in terminology of penalty function.

3. Chaos Flow Control with Penalty Function

3.1.

Consider a finite digraph

$$\mathbf{\Gamma = (V(\Gamma), E(\Gamma), v^+, v^-, \varphi)}, \qquad (7)$$

where $E(\Gamma)$ is the arc-set, $V(\Gamma)$ is the vertex-set containing a source $v^+ \in V(\Gamma)$ and hole $v^- \in V(\Gamma)$, and $\varphi : E(\Gamma) \longrightarrow \mathbb{R}^+$ is the function defining the capacity of arcs.

We can digraph Γ (7) in form a chaotics $\mathscr{H} = (A : C_i, i\in\mathbb{N}(n))$, where $A = V(\Gamma)$, $C_i = E(\Gamma)$, and $\varphi : E(\Gamma) \longrightarrow \mathbb{R}^+$ is a penalty function, $[i]\in\mathbb{N}(n)$ is the controllers. Preference $\succcurlyeq_i$ give the penalty functions. All controllers is active. The law of control be determined next graphical constructions.

Let

$$\mathscr{P} = \{\mathbf{P \subset V(\Gamma) : v^+ \in P, v^- \notin P}\}.$$

For $P\in\mathscr{P}$, we refer to

$$R(P) = \{e\in E(\Gamma) : \partial^+ e \in P, \partial^- e \notin P\}$$

as the **cut** corresponding to **P** and define its **value** of a penalty function by

$$\varphi(P) = \sum_{i=1}^{r} \phi(\mathbf{e_i}),\ e_i\in R(P),\ r = |R(P)|.$$

A **flow** in **Γ** is a function

$$\Theta : E(\Gamma) \longrightarrow \mathbb{R}^+$$

that satisfies capacity condition:

$$0 \leq \Theta(e) \leq \varphi(e)$$

for each $\mathbf{e\in E(\Gamma)}$ and the conservation condition:

$$\Theta(\delta^+ v) = \Theta(\delta^- v)$$

at each vertex $\mathbf{v\in V(\Gamma)}$ distinct from $\mathbf{e^+}$ and $\mathbf{e^-}$, where

$$\Theta(\delta^+ v) = \sum_{\forall e\in\delta^+ v} \Theta(\mathbf{e}) \text{ and } \Theta(\delta^- v) = \sum_{\forall e\in\delta^- v} \Theta(\mathbf{e}).$$

A **chaos flow control of digraph Γ** with the penalty function $\varphi : \mathbf{E(\Gamma)} \longrightarrow \mathbb{R}^+$ is maximization the value of flow Θ.

Proposition 2. [See 5]. The chaos flow control of digraph **Γ** with the penalty function $\varphi : \mathbf{E(\Gamma)} \longrightarrow \mathbb{R}^+$ is equal to the minimum capacity of a cut.

Corollary 2.1. Efficient algorithms of complexity such as $\mathbf{O(|V(\Gamma)|^3)}$ are known for finding a maximum flow.

The proof and the algorithm are found in [1].

3.2.

The basic definitions of the terms pertaining to flows control in general chaotic are as follows.

Let $\mathfrak{I} = (\mathbf{A},\mathbf{C})$ be finite chaotic with the groundset $\mathbf{A} = \{\mathbf{a_0,a_1,\ldots,a_m}\}$ and the cycles $\mathbf{C} = \{\mathbf{C_1,C_2,\ldots,C_d}\} \subset \mathbf{2^A}$. $\mathbf{a_0 \in A}$ and is called the flows with input in $\mathbf{a_0}$ of $\mathfrak{I}$. Given $\mathbf{C(a_0)} \subseteq \mathbf{C}$, $\mathbf{C(a_0)} = \{\alpha_j \in \mathbf{C} : \forall \alpha_j \ni \mathbf{a_0}\}$, $\mathbf{j = [1,r]}$. And given

$$\mathbf{V} = \{\mathbf{v_k} \in \mathbb{Q}^+ : \mathbf{k = [1,m]}\},$$

where $\mathbf{v_k}$ is called **weight** of the element $\mathbf{a_k \in A}$.

There is a standard a chaotic flow control problem. Really, the chaotic $\mathfrak{I} = (\mathbf{A},\mathbf{C})$ is the controlled chaos. **A** is the position-set, $\mathbf{[i] \in \mathbb{N}(d)}$ is the controllers, $\mathbf{C_i}$ is the territory of the controller **[i]**, and $\mathbf{Z = \{[1], \ldots, [d]\}}$ is the control-brigade. The law of control and penalty functions be determined next chaotic constructions.

Further, given the matrix $\mathbf{M} = [\tau_{ij}]_{n \times r}$, where $\tau_{ij} = \mathbf{1}$ if $\mathbf{a_i} \in \alpha_j$. Since $\mathbf{a_i} \notin \alpha_j$, we have $\tau_{ij} = \mathbf{0}$. The matrix **M** is called **flow-matrix across the cycles C(a₀)**. Finally, define the flows $\mathcal{F}_{\mathfrak{I}}$ of $\mathfrak{I}$ by

$$(\mathbf{p_1,p_2,\ldots,p_r}),\ \mathbf{p_i} \in \mathbb{Q}^+,$$

where

$$\sum_{j=1}^{r} \tau_{ij}\mathbf{p_j} \le \mathbf{v_i},\ \mathbf{i = [1,d]}.$$

$\mathcal{V}_{\mathfrak{I}} = \sum_{j=1}^{r} \mathbf{p_j}$ is called value of flows $\mathcal{F}_{\mathfrak{I}}$. (The penalty function!)

Examples. A blood flow, a limphe flow, a toxic flux, geotectonic flow, a peniciline propagation and other are examples for flow in chaos.

Figure 2. Anatoly Fomenko. From the flows

Finally we assume that

$$(\mathbf{x_1^0}, \ldots, \mathbf{x_r^0})$$

is the solution of the problem

$$\mathrm{P}(\mathfrak{I}) = \sum_{j=1}^{r} \mathbf{x_j} \longrightarrow \max$$

$$\sum_{j=1}^{r} \tau_{ij}\mathbf{x_j} \le \mathbf{v_i},\ \mathbf{i = [1,d]}.$$

The vector $(\mathbf{x_1^0}, \ldots, \mathbf{x_r^0})$ is a maximum $\mathfrak{I}$-flow in the presence of weights $\mathbf{V} = \{\mathbf{v_i} \in \mathbb{Q}^+ : \mathbf{i=[1,d]}\}$ and flows with input in $\mathbf{a_0 \in A}$. If $\mathbf{c_k}$ is cycle of $\mathfrak{I}$ which contains $\mathbf{a_0}$ then by the capacity $\mathbf{f(c_k)}$ of $\mathbf{c_k}$ (with respect to b) we mean

$$\mathbf{f(c_k)} = \sum_{i|a_i \in c_k} \mathbf{v_i}.$$

We say that the chaotic $\mathfrak{I} = (b)$ is called a **regular** if for each $\mathbf{a_0 \in A}$ which is not a loop of $\mathfrak{I}$

and for any set of capacities $\mathbf{V}=\{\mathbf{v_1}, \ldots, \mathbf{v_d}\}$ the value of the maximum $\mathfrak{J}$-flow equals the minimum capacity $\mathbf{f^{min}(c_k)}$ of $\mathbf{a_0}$, i.e.

$$\mathbf{P^{max}(\mathfrak{J}) = f^{min}(c_k)}. \tag{8}$$

The figure 2 is the illustration of the chaotic-flow.

Next, we are now in a position to state the problem of chaotic theory.

Problem JULIA. Let $\mathfrak{J} = (\mathbf{A,C})$ is a finite chaotic. Where $\mathfrak{J}$ is a regular chaotic?

Theorem 3. Let $\mathfrak{J} = (\mathbf{A, C})$ is a finite chaotic. $\mathbf{a_0 \in A}$ is not a loop and for any set of capacities $\mathbf{V}=\{\mathbf{v_1}, \ldots, \mathbf{v_d}\}$ $(\forall \mathbf{v_i} \geq \mathbf{0})$ is the value of the maximum $\mathfrak{J}$-flow equivalent the minimum capacity $\mathbf{f^{min}(c_k)}$ of $\mathbf{a_0}$, i.e.

$$\mathbf{P^{max}(\mathfrak{J}) \leq f^{min}(c_k)}.$$

The proof and the algorithm are found in [2].

4. Control Strategy against External Controller

4.1.

A **field operator** of the **field** $\mathbf{S \subseteq A}$ is a map $\varphi : \mathbf{S} \longrightarrow \mathbf{A}$. Let

$$\mathbf{\Omega} : \mathrm{A} \xrightarrow{\;\;¤\;\;} \mathrm{A}$$

is a law of the control for the chaotic $\mathbb{H} = (\mathbf{A} : \mathbf{C_i}, \mathbf{i \in \mathbb{N}(n)})$. A **strategy** of a controller **[i]** is the field operator

$$\psi_\mathbf{i} : \mathbf{C_i \setminus C_0} \longrightarrow \mathbf{\Omega} \, (\mathbf{C_i \setminus C_0}), \tag{9}$$

where $\mathbf{C_0} = (\boldsymbol{\alpha} : \mathbf{\Omega}(\boldsymbol{\alpha}) = \varnothing)$ and $\mathbf{\Omega}\,(\mathbf{C_0}) \cap \mathbf{C_i} = \varnothing$. The controller **[i]** will be considered to have a fixed the strategy $\psi_\mathbf{i}$. Let a **row string** is

$$\Psi \overset{\text{def}}{=} (\psi_1, \psi_2, \ldots, \psi_n)$$

such that $\psi_\mathbf{i}$ are the strategy (9). The multi map

$$\Psi : \mathbf{A \setminus C_0} \xrightarrow{\;\;¤\;\;} \mathbf{A} \tag{10}$$

is defined by requiring Ψ to be the field operator $\psi_\mathbf{i}$ on $\mathbf{C_i \setminus C_0}$, i.e. $\Psi(\boldsymbol{\alpha}) = \psi_\mathbf{i}(\boldsymbol{\alpha})$, where $\boldsymbol{\alpha} \in \mathbf{C_i \setminus C_0}$.

Further let $\mathbf{R} = (\mathbf{i_1}, \mathbf{i_2}, \ldots, \mathbf{i_r})$ are the index of an active controllers, $\mathbf{P} = \mathbb{N}(\mathbf{n})\setminus\mathbf{R} = (\mathbf{j_1}, \mathbf{j_2}, \ldots, \mathbf{j_{n-r}})$ are the index of an passive controllers for control-brigade

$$\mathbf{Z} = \{[1], \ldots, [n]\}.$$

$\mathbf{Z} = \{[\mathbf{i_1}], \ldots, [\mathbf{i_r}], [\mathbf{j_1}] \ldots, [\mathbf{j_{n-r}}]\} = \{\mathbf{Z_R}, \mathbf{Z_P}\}$. A multi map Ψ (10) is called a **strategy** of a control-brigade **Z**.

Theorem 4. A strategy Ψ is defined uniquely of the control for the chaotic $\mathbb{H}$ if $\mathbf{C_0}$ be fixed.

Proof. The proof of the theorem 4 is similar.

4.2.

Suppose a row string

$$\{\Psi\} = \{\psi_1, \ldots, \psi_\mathbf{k}\}$$

is a set **traversed** the control position (see (6)), where **k** is a depth of ditree $\mathsf{T}^{\succ}$. Further assume that a row string

$$\{\Psi^\circ\} = \{\Psi^\circ_1, \ldots, \Psi^\circ_{\mathbf{k}^\circ}\}$$

other a set traversed the control position. We call the strategy **Ψ** ***preferable*** for a controller **[i]** of the strategy Ψ° if for $\forall\alpha\in C_i$ **Ψ (α)** occur $\Psi^{\circ}(\alpha)$ ($\Psi(\alpha) \succcurlyeq_i \Psi^{\circ}(\alpha)$) and is denoted by

$$\{\Psi\} \succcurlyeq_i \{\Psi^{\circ}\}. \qquad (11)$$

Theorem 5. The relation $\succcurlyeq_i$ (11) is a preference in a set of strategy of the control for the chaotic $\mathbb{H}$.

The proof is straightforward.

The relation $\succcurlyeq_i$ (11) is called a **strategy preference**.

4.3.

A strategy preference $\succcurlyeq_i$ of controller **[i]** be written in terminology of penalty function **f : {{Ψ}}** ⟶ ℝ, where **{{Ψ}}** is the set all strategy (10).

$$f_i^+(\Psi) = \sup (f_i(x) : x\{\Psi\}) \text{ if } [i]\in Z_R, \qquad (12)$$

$$f_i^-(\Psi) = \inf (f_i(x) : x\{\Psi\}) \text{ if } [i]\in Z_P. \qquad (13)$$

In terminology of penalty function the strategy preferance (11) is rephrased

$$f(\Psi) \geq f(\Psi^{\circ}). \qquad (14)$$

4.4.

Let ψ_i is an arbitrary strategy of a controller **[i]** and $\Psi_{N(n)\backslash i}$ is a control strategy Ψ without strategy of the controller **[i]**. A strategy $\Psi^* = \{\Psi_1^*, \ldots, \Psi_n^*\}$ is called **control strategy against external controller [i]** if

$$\Psi^* \succcurlyeq_i (\psi_i, \Psi^*_{N(n)\backslash i}), \qquad (15)$$

where **[i]∈Z**.

By (15) is meant the controller **[i]** there is nothing to prevent of every remaining controllers.

Further a strategy **Ψ*** is called a **safety control strategy** if the strategy preference (15) be realized for all controller **[i]∈Z**.

Finally a strategy **Ψ*** is called a **antiterrorist control strategy** if the strategy preference (15) be realized for all exterior controller **∀[n+1]∈Z**, see figure 3.

The algorithms of construction safety&antiterrorist control strategy for chaotic will be object of next paper.

Figure 3. Ivan Nevidomyj. The Penetration

5. Direct Algorithm of Passive Control on ℤ×ℤ (Algorithm 2)

In sections 4-6 will be concerned of a problems control with one passive controller $[1]^-$, position-set of control **A** ⊂ ℤ×ℤ, controlled chaotic **H = (A, Ω)**, $\Omega \subseteq 2^A$. This problem is well-know was named for the monitoring infecting zone [1] — [3].

Let the subset **B ⊆ A** is a beginning infected zone with a boundary **L** and **L** is a cycle curve without an intersection. **L** is called a **boundary zone B**. **ℑ = (A, Ω, B)** is a **beginning infect front**. Further, we may applicable the algorithm 1.

Suppose **S(A) ⊇ A** is a minimal sphere with the center $O_A = (i_o, j_o) \in B$,

$$\pi: \mathbf{A} \longrightarrow \{+1, -1\}, \tag{16}$$

be the function that takes each two cells $\mathbf{k} = (\mathbf{i_k}, \mathbf{j_k})$ and $\mathbf{m} = (\mathbf{i_m}, \mathbf{j_m})$ to

1) $\pi(\mathbf{k}, \mathbf{m}) = +1$, if $|\mathbf{i_m} - \mathbf{i_o}| > |\mathbf{i_k} - \mathbf{i_o}|$ or $|\mathbf{j_m} - \mathbf{j_o}| > |\mathbf{j_k} - \mathbf{j_o}|$;
2) $\pi(\mathbf{k}, \mathbf{m}) = -1$ on the other case.

S(A) is called a restriction sphere.

By definition the map (16) is a strategy of a controller **[1]**.

Correctly are the following theorem.

Theorem 6. [See 5]. Let a finite ground-set $\mathbf{A} \subseteq \mathbb{Z}\times\mathbb{Z}$ is a restriction sphere **S(A)** is a finite ground-set **A** and $\mathbf{\Gamma} = (\mathbf{V}, \mathbf{E})$ is a finite graph, where the vertex-set $\mathbf{V} = \mathbf{A}$ and edge-set $\mathbf{E} = \{\mathbf{e} \in \mathbf{B}: \mathbf{e} \in \{\forall(\mathbf{i},\mathbf{j}) \in \mathbf{V}: (\mathbf{i}\text{-}1,\mathbf{j}), (\mathbf{i},\mathbf{j}\text{-}1), (\mathbf{i}+1,\mathbf{j}), (\mathbf{i},\mathbf{j}+1)\}\}$, $\mathbb{N}(\mathbf{\Gamma})$ is the inverse function for the algorithm 1 on a graph **Γ**. Then we have

$$\mathbb{N}(\mathbf{\Gamma}) = \sum_{\pi}{}' \theta^{\mu(\mathbf{E}_\pi^+)-\mu(\mathbf{E}_\pi^-)}, \tag{17}$$

where $\mathbf{E}_\pi^+$ to be the set of edges $(\mathbf{d}, \mathbf{r})$ of **Γ** such that $\pi_\mathbf{d}\pi_\mathbf{r} = \mathbf{1}$ and $\mathbf{E}_\pi^-$ be the remaining edges of **Γ**, here $\pi_\mathbf{d}\pi_\mathbf{r} = \mathbf{1}$ if the diedge $(\mathbf{d}, \mathbf{r})$ is "**i•→j**", $\theta = \mathbf{e}^\mathbf{I}$, here **I** is a number of the Algorithm 1 iterations and **π** is strategy of controller (16).

Corollary 6.1. [See 9]. Suppose $\mathbf{N^o}$ is an iteration number of algorithm 1 on graph **Γ**; then $\mathbf{N^o} \le \mu(\mathbb{N}(\mathbf{\Gamma}))$.

Corollary 6.2.. The algorithm 1 on $\mathbb{Z}\times\mathbb{Z}$ has big computability complexity.

6. Direct Algorithm of single-center infection on $\mathbb{Z}\times\mathbb{Z}$ (Algorithm 3)

Suppose conditions of theorem 6 being satisfied. The subset $\mathbf{B} \subseteq \mathbf{A}$ is a beginning infected zone with the boundary zone **L**. Let $\mathbf{L} \subseteq \mathbf{B}$ and the subset $\mathbf{A}^\heartsuit \subseteq \mathbf{A/B} \subseteq \mathbb{Z}\times\mathbb{Z}$ are **infection-screened cells**. Infection screened cells are marked the symbol "**♥**", see fig.5. The boundary zone B contains an **infection centre $\mathbf{O_A} \in \mathbf{B}$**. Any cell $\mathbf{c} \in \mathbf{B}$ is called **active** if $\{\mathbf{c}\} \cap \mathbf{L} \ne \varnothing$ and the active cells are **starting points** of algorithm 3. The cells

$$\mathbf{c_{ij}} = \{(\mathbf{i} \pm 0;1\ ,\ \mathbf{j} \pm 0;1) \ne (\mathbf{i},\mathbf{j})\} \in \mathbf{A}$$

are called neighbouring cells of cell (i,j)∈A. Any cell $c \in A/(B \cup A^\heartsuit)$ is called a freedom cell (f-cell). For any cells $c_1 = (i,j)$, $c_2 = (k,l) \in A$ there exists a distance

$$\mathbf{d}(\mathbf{c_1}, \mathbf{c_2}) = \sqrt{(\mathbf{i}-\mathbf{k})^2 + (\mathbf{j}-\mathbf{l})^2}.$$

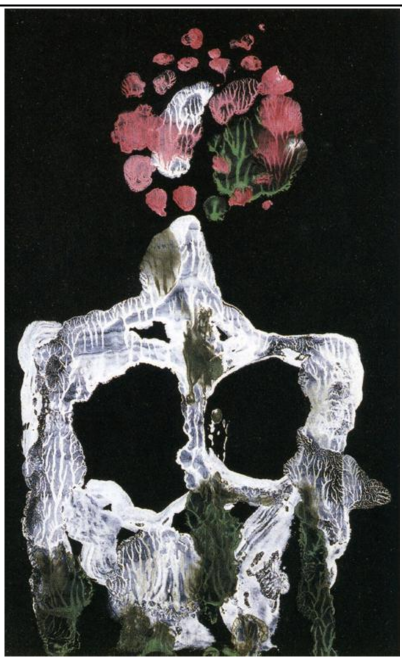


Figure 4. Salvatore Dali. The Pandemia

Algorithm 3

(**RI**) We get $\mathbf{L_1} = \mathbf{L}$.

(**Z**) An active cell $\mathbf{m} \in \mathbf{L}$ is said to be **initial** if $\mathbf{d}(\mathbf{m}, \mathbf{O_A}) \xrightarrow[\mathbf{L}]{} \mathbf{min}$. Let $\mathbf{C_m} = \{\mathbf{m_1}, \mathbf{m_2}, \mathbf{m_3}\}$ be the **f**-cells, where $\mu(\mathbf{m_i} \cap \mathbf{m}) = \mathbf{2}$, and $\mathbf{m^*}$ be the cell of $\mathbf{C_m}$ such that $\mathbf{m^*}$ have the maximal number $\mathbf{n(m^*)}$ of neighbouring active cells. $\mathbf{m^*}$ stand of the active cell. $\mathbf{L_1} := \mathbf{L_1}/\{\mathbf{m}\}$

∪**{m*}**. If **m*** not exist, then $\mathbf{L_1}$:= $\mathbf{L_1}$.

The rule (**Z**) by repeats **μ(L) – 1** time in the hour-hand direction.

If $\mathbf{L_1} \neq \mathbf{L}$, then $\mathbf{L} = \mathbf{L_1}$, we add to $\mathbf{A}^{♥}$ the new **f**-cell and go to (**RI**). Finally, if $\mathbf{L_1} = \mathbf{L}$, then the algorithm 3 is **stop**.

Example. In Fig.5 the new active cells are marked the sign "■", the active cells are marked the sign "×", and the f-cells are shown as "♥".

Theorem 7. [See 9]. The algorithm 3 has the computation complexity $\mathbf{O(m^2)}$, where $\mathbf{m = \mu(L)}$.

Corollary 7.1. The algorithm 3 is effective to solution of real problems for computational viral extension.

Corollary 7.2. Let $\mathbf{B} = \mathbf{O_A} = \mathbf{L}$; then $\mathbf{P} = \mathbf{n_a} / \mathbf{n_f} \approx 0.57$, where $\mathbf{n_a}$ is the number of the new active cells and $\mathbf{n_f}$ is the number of the **f**-cells. Using the algorithm 3 computa-tional experiments we obtain $\mathbf{n_a}$ and $\mathbf{n_f}$.

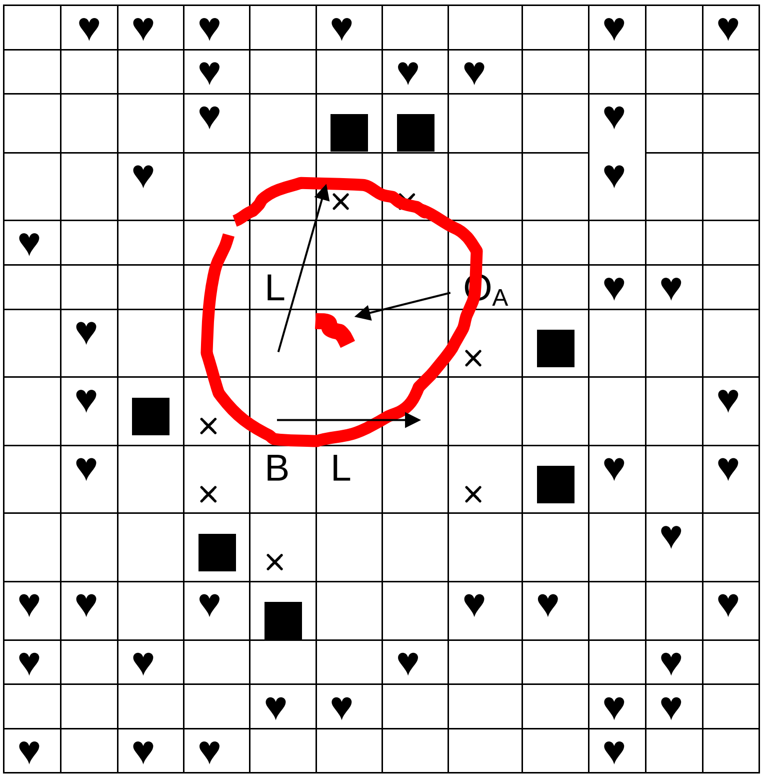


Figure 5.

Hypothesis. The number **P** is a critical probability, see (4), of single-center infection on ℤ×ℤ.

7. Inverse Algorithm of Single-Center Infection

Suppose conditions of section 6 being satisfied. Let we have $\mathbf{B} = \mathbf{O_A} = \mathbf{L}$ of an initial infection. Furthermore, **B* ⊆A** is an infected zone before starting inverse algorithm.

Let **H ⊆ A / B*** is fixed subset of **f**-cells.

Theorem 8. Let **A ⊆** ℤ×ℤ is finite the ground-set, **S(A)** is a a restriction sphere, and **π** is function (16). Further, let **Γ = (V, E)** is a finite graph, where the vertex-set **V = A** and edge-set **E = {e∈B: e ∈{∀(i,j)∈V: (i-1,j), (i,j-1), (i+1,j), (i,j+1)}}**, **ℕ(Γ)** is the inverse function for the Algorithm 3 on a graph **Γ**. Then we have

$$\mathbb{N}(\Gamma) = (\theta + \theta^{-1}) \times \mathbb{N}(\Gamma \propto \{e\}) - \theta^{-1} \times \mathbb{N}(\Gamma \div \{e\}), \quad (18)$$

where **Γ∝{e}** be the graph obtained by deleting an edge **e** from **Γ**, **Γ÷{e}** be the graph obtained by deleting an edge e and then identifying its end points, $\theta = e^I$, here **I** is a number of the Algorithm 3 iterations.

Theorem 9.

$$I^* \le (\mu(A) - \mu(H))^2,$$

where **I*** is a number of the Algorithm 4 iterations. **A** is a groundset and **H** is the subset of all protected cells.

Our main result is the following.

Theorem 10. There exist the algorithm 4 of effective solution of the inverse problems for computational viral extension.

The proof theorems 4-6 are in [7]-[12] and the algorithm 4 listing is in [9].

Corollary 10.1. Using Algorithm 4, we get the coordinates:

$$O_A = (i_0, j_0), \quad (19)$$

where O_A is an infection center.

Remark. The coordinates (19) are a dream of antiterrorist organizations.

Figure 6. S. Grosz. The Passive Conductor

R e f e r e n c e s :

1. V.V.Gritsak (Valery V. Gritsak–Groener), V.S. Michalevitsh. Direct & Inverse Algorithms of Viruses Spreading. USSR Fond of Priority Directions, VINITI, Moskva, 1984.
2. V.V.Gritsak (Valery V. Gritsak–Groener), Alexander S. Davydov. Algorithms& Metods for Computational of Inverse Electromagnetic Scattering. Energia, Moskva.1984.
3. Valery V.Gritsak–Groener, Julia Gritsak–Groener, Hamid R. Arabnia, Algorithms for Computational Viral Extension Direct & Inverse Problems, 2007 International Conference on Scientific Computing, Las Vegas, Nevada, USA Jule, 2007, pp.284-295.
4. V.V.Gritsak (Valery V. Gritsak–Groener). Proc. Nat. Acad. Sci. Ukraine SSR, ser. A, N6(1990)68.
5. V.V.Gritsak (Valery V. Gritsak–Groener), Kozlov A. I. Methods of geophysical tapes uniform randomness, Moskva, ISTM, 1993.
6. Valery V. Gritsak–Groener, O.MacLarty. Proc. Nat. Acad. Sci. Ukraine, N6(2002)62.
7. Valery V. Gritsak–Groener .Theory of Finite Chaotic. M., SLU,1997.
8. Valery V.Gritsak-Groener., Julia Gritsak-Groener, ARTS COMBINATORIA. NTU “HPU”, Charkiv, second edit, 2008.
9. Wilhelm Groener (Valery V.Gritsak–Groener), Fundamental of Mathematical Cybernetics. MIT Press, 2004.
10. Valery V.Gritsak–Groener, Fundamental of Mathematical Cybernetics, MIT Press, 2006.
11. Valery V.Gritsak–Groener, Julia Gritsak–Groener, Hamid R. Arabnia, Modern Mathematical Biology, München, MIT Press, 2006.
12. Valery V.Gritsak–Groener, Julia Gritsak–Groener, Algorithms of Control for Combinatorial Chaos. Chaos 4., PHISICA, 2012.

Статья поступила в редакцию 07.04.2012 г.

Гритсак-Грёнер В.В., Гритсак-Грёнер Ю.

Стратегическое управление хаосом и обратные задачи

Мы продолжаем вводить новые понятия контроля и управления хаосом. Мы вводим, также понятие стратегии управления хаотическими структурами. В разделе 2 дан краткий обзор стандартных фактов о контроле хаотиков. В разделе 3 мы даём некоторые базисные факты о контроле (управлении) хаотичных потоков со штрафной функцией. В разделе 4 мы снова вводим новые понятия стратегии контроля, которые позволяют полностью контролировать хаос, даже в ситуациях, когда один из контролёров из бригады контролёров предательски начинает менять свою стратегию или, более того, появляется один из внешних контролёров со своей стратегией. В обоих случаях, наша стратегия не позволяет предателю или террористу (так, естественно мы называем этих контролёров) получить, какую-нибудь выгоду. Дальше, в разделе 5, мы рассматриваем конкретный случай контроля, когда множество контроля имеет координатную целочисленную сетку $\mathbf{A} \subset \mathbb{Z}\times\mathbb{Z}$ и у нас один пассивный контролёр. Задача контроля хаоса, несмотря на ограничения, довольно распространенная. В другой терминологии она называется задачей мониторинга на координатной сетке. Например, к таким относится мониторинг распространения вирусной эпидемии или распространения последствий стихийного бедствия или техногенной катастрофы. В разделах 5–7 мы её полностью алгоритмически решаем. В секции 5 мы приводим прямой алгоритм её решения. В разделе 6 приводится алгоритм решения в случае известного центра распространения хаоса. (Например, для распространения нуклидов из Чернобыльской атомной электростанции). В обоих случаях предусматривается стратегия протии вмешательства предателей и террористов. Наконец в разделе 7 приводится знаменитый алгоритм, принадлежащий первому автору, решения обратной задачи мониторинга. Другими словам, вычисляются координаты центра распространения беды.

Ключевые слова: хаос, хаотик, алгоритм