SOCIAL PROCESS MODELING

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CLASSICAL MATHEMATICAL SOCIOMETRY III. GRAPHIC METHODS OF ORDERED SOCIUM

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In the article mathematical methods of social decision-making are developed, when the ordering of solutions is given by an arbitrary finite graph. For example: ordered, weighed and painted multitudes. The statement of the problem is but has never been considered in mathematical sociometry. Developed in our articles mathematical methods for direct and duality mathematical problems of the tasks of a single executor under the control of a socium α under the scheme of an arbitrary finite graph **G** allow algorithmically solving any real problem of a sociometric planning.

Key words: socium, multi digraphs, color, order.

1. Elements of the theory graphs

Graphs (from the word graph, graffiti) are a convenient, clear, geometric representation of binary relations, which we have already considered in previous articles. The clarity of the graphs contributed to their great popularity among humanitarian, sociological and economic applications.

First formal definitions.

Definition 1.1.

A simple graph A simple graph Γ is an ordered pair

$$\Gamma = (\mathbf{V}(\Gamma), \mathbf{E}(\Gamma)), \tag{1.1}$$

where **V(Γ)** is a non-empty set, and **E(Γ)** is the binary relation on the set **V(Γ)**, and other subwords of the Cartesian product **V(Γ)** × **V(Γ)**. The elements of the set **V(Γ)** will be called the **vertices** of the graph Γ, and the elements of the set **E(Γ)** will be called the **edges** of the graph Γ.

Example 1.1.

Figure 1 exhibit the simple graph Γ. *Its vertices are set*

$$\mathbf{V}(\Gamma) = \{v_1, v_2, v_3, v_4, v_5\}.$$

The elements of **V(Γ)** *marked with red circles in figure 1. The edges of* Γ *are set*

$\mathbf{E}(\Gamma) = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11}, e_{12}, e_{13}, e_{14}, e_{15}\}$.

The elements of **E(Γ)** marked with blue lines. The edge can connect two vertices, then the corresponding line will be straight (e.g. $\mathbf{e_4}$), but may connect the vertex itself with itself ($\mathbf{e_{11}}$, for example), then the line will be the loop above the corresponding vertex.

If the graph Γ is interpreted as an economic or political relationship between members of a group of 5 members. Then, the vertices Γ will be members of this group, and the edges are pairwise. Loops of the graph Γ can be interpreted as political, or economic self-consciousness.

In the general case, you can have as many arbitrary edges between two vertices, so we will have the following definition.

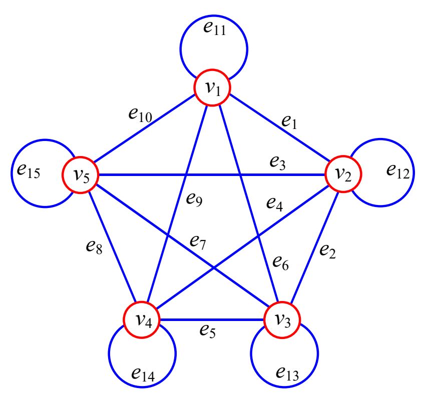


Fig. 1.1. A complete 5-vertex simple graph Γ.

Definition 1.2.

A **general graph** or simply **a graph G** is an ordered pair

$$\mathbf{G} = (\mathbf{V(G)}, \mathbf{E(G)}), \quad (1.2)$$

where **V(G)** is the set of its vertices, and the set **E(G)** of its edges can be represented as the union of

$$\mathbf{E(G)} = \mathbf{E}_1(\Gamma_1) \cup \ldots \cup \mathbf{E}_n(\Gamma_n)$$

of the edges $\mathbf{E}_i(\Gamma_i)$, $\mathbf{i = [1, n]}$ of some simple graphs $\Gamma_i = (\mathbf{V(G)}, \mathbf{E}_i(\Gamma_i))$.

Example 1.2.

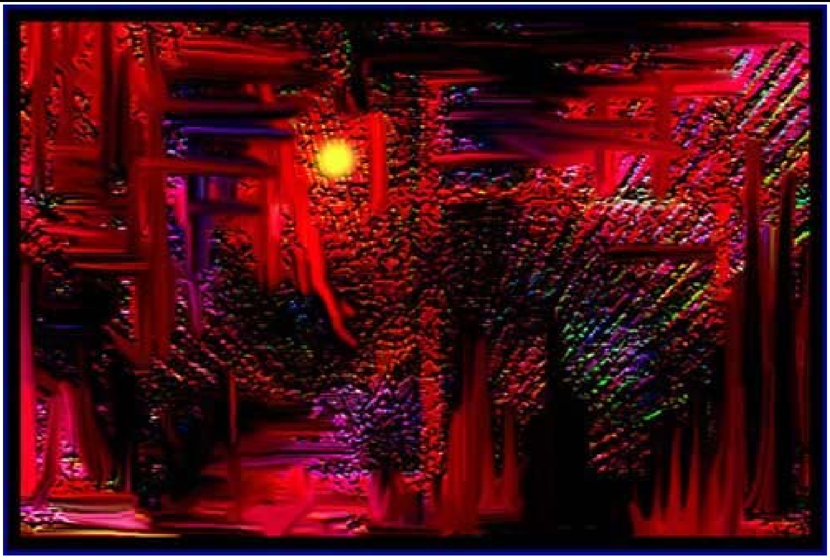
In Figure 1.2 (in picture 1) , is depicted the general graph **G**. Its vertices will be the set to

$$\mathbf{V(G)} = \{v_1,v_2,v_3,v_4,v_5\},$$

and the elements marked with red circles, and its edges will be the set

$$\mathbf{E(G)} = \{e_1,e_2,e_3,e_4,e_5,e_6,e_7,e_8,e_9,e_{10},e_{11},e_{12},e_{13},e_{14},e_{15},e_{16},e_{17},e_{18},e_{19}, e_{20}\}.$$

The edges are marked with blue lines. We see that there may be a several edges between the two vertices (for example, between the vertices v_1 and the vertices v_2 there will be the three edges - e_1, e_2 and e_3). The definition of the graph does not forbid us to have a few loops over an one vertex, but our main interpretation of the loop, as self-consciousness leads in such cases to clinical and psychiatric cases of members of a society with a split, trivial, etc. consciousness, conscience. Therefore, we do not indicate in the figure 1.2 the multiplayer over the vertices. The multiedges between two different a vertices are well interpreted as the multiplicity of relations between a member or an organizations of a society, which is often found in real situations.



Pict. 1. I .Nevidomyj. The red graph.

Further, we introduce a convenient graphic terminology, which we will use throughout our book and beyond.

The pair

$$\Gamma = (\mathbf{V}(\Gamma), \mathbf{E}(\Gamma)) \quad (1.3)$$

is a graph. If $\mathbf{e = (v_1, v_2)}$ is its edge, then the vertices $\mathbf{v_1}$ and $\mathbf{v_2}$ are called a **ends** of the edge **e**. In this case, the vertices $\mathbf{v_1}$ and $\mathbf{v_2}$ are by an **incindent** edge **e**. A set of two or more edges having the same ends are called a **parallel** to each other. The edge of both ends of the coincident is called a **loop**. The two edges $\mathbf{e_1 = (v_1, v)}$ and $\mathbf{e_2 = (v, v_3)}$ are called the **adjacent** when they have the common vertex (in this case, the vertex **v**). The vertices $\mathbf{v_1}$ and $\mathbf{v_2}$ are adjacent to each other if there is an edge $\mathbf{e = (v_1, v_2)}$. The number of edges of the incipient vertex $v \in \mathbf{V}(\Gamma)$ is called a **degree** of the vertex **v** and is denoted by **st (v)**. The vertex $\mathbf{v_0}$ whose degree is equal to **0** ($\mathbf{st(v_0) = 0}$) is called an **isolated**. The graph $\Gamma = (\mathbf{V}(\Gamma), \mathbf{E}(\Gamma))$ will be considered **finite** if the set of it's the vertices $\mathbf{V}(\Gamma)$ and the edges $\mathbf{E}(\Gamma)$ are finite simultaneously. Denote by $\mathbf{n}_\Gamma\mathbf{(V)}$ and $\mathbf{n}_\Gamma\mathbf{(E)}$ the number of vertices and edges of the graph Γ, respectively. The number $\mathbf{n}_\Gamma\mathbf{(V)}$ will be called **order** of the graph Γ, and the number $\mathbf{n}_\Gamma\mathbf{(E)}$ is a **volume** of the graph Γ.

A simple graph Γ_0 is called **complete** if each pair of the vertices of Γ_0 is adjacent, for example, there is a graph from figure 1. A **track L** in a graph Γ (not necessarily simple) is alternating (once vertex, then edge, then vertex, etc.) sequence of the form:

$$v_1 e_1 v_2 e_1 v_3 \ldots e_n v_{n+1}, \qquad (1.4)$$

the vertices and incident the edges, which begins with the vertex $\mathbf{v_1}$ and ends with the vertex $\mathbf{v_{n+1}}$. In addition, each of two vertices $\mathbf{v_i}$ and $\mathbf{v_{i+1}}$, **i = [1,n]**, are finite for the edge $\mathbf{e_i}$. The vertex $\mathbf{v_1}$ is called an **input**, and the vertex $\mathbf{v_{n+1}}$ is an **output** of the track **L**. A track **S** is called by a **path** if all its vertices, except for the possible input $\mathbf{v_1}$ and output $\mathbf{v_{n+1}}$, are pairwise different. If the input and exit of a path **S** coincide, then **S** is called a **loop**. A simple graph $\mathbf{\Gamma_0}$, which does not have any cycle among its vertices and edges, is called a **tree**. A graph **Γ** is called **connected** if for any of its two vertices $\mathbf{v_1}$ and $\mathbf{v_2}$ there is a track **L** in the graph **Γ** such that $\mathbf{v_1}$ and $\mathbf{v_2}$ are its, respectively, an input and an output of **L**.

Example 1.3.

In figure 1.2, a general graph **G** is depicted. The ends of the edges $\mathbf{e_1}$ are the vertices $\mathbf{v_1}$ and $\mathbf{v_2}$. The vertices $\mathbf{v_1}$ and $\mathbf{v_2}$ will be incident edges $\mathbf{e_1}$. The edges $\mathbf{e_1}$, $\mathbf{e_2}$ and $\mathbf{e_3}$ are pairwise parallel to each other. The edge $\mathbf{e_{20}}$ is the loop. The edges $\mathbf{e_1}$ and $\mathbf{e_7}$ are adjacent. The vertices $\mathbf{v_1}$ and $\mathbf{v_3}$ are adjacent. The degree of vertex $\mathbf{v_2}$ is **9**, or in our notation **st**(v_2) = 9. Isolated vertices in the graph **G** are not. Graph **G** is finite. $\mathbf{n_G(V)}$ = 5, and $\mathbf{n_G(E)}$ = 20. In other words, the graph **G** has order **5**, and the volume **20**.

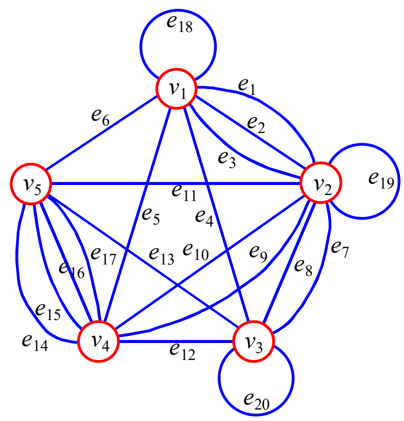


Fig. 1.2.

Figure 1.1 depicts the simple graph **Γ**. The graph **Γ** is complete. The track **L** will, for example, be the sequence $\mathbf{v_1}$ $\mathbf{e_2}$ $\mathbf{v_2}$ $\mathbf{e_{12}}$ $\mathbf{v_2}$. But **L** will not be a path. By the path of **S** will be the sequence $\mathbf{v_1}$ $\mathbf{e_{10}}$ $\mathbf{v_5}$ $\mathbf{e_7}$ $\mathbf{v_3}$. But **S** will not a cycle. The cycle **C** will be the sequence $\mathbf{v_1}$ $\mathbf{e_6}$ $\mathbf{v_3}$ $\mathbf{e_5}$ $\mathbf{v_5}$ $\mathbf{e_9}$ $\mathbf{v_1}$. Both graphs of figures 1.1 and 1.2 are linked.

It is easy for readers to independently come up with good examples of the terminology of a life. For example, the cycle will be the route of an unfortunate pensioner to various bureaucratic offices for an increase of 5 hryvnias of his miserable pension. The vertices here will be the offices, and it's the edges the track from one office to another office.

The geometric or the graphic representation of graphs is very convenient on papers and on the computer displays. For computer calculations, for example finding the optimal solutions for the graph control models, thegraph information must be represented as numerical arrays. For this purpose, an information about graphs is convenient to represent them with the incidence matrix.

Let **G** be a finite graph with **n** vertices, which are denoted by **1,2,..., n**. A **incidence matrix** of the graph **G**, which we denote $M_n^n(G)$, is the matrix dimension **n × n**, in which an integer on the intersection of the **i** line and the **j** column is equal to the number **k** of edges between the **i**-th and **j**-th vertices.

	Col. 1	**Col. 2**	**Col. 3**	**Col. 4**	**Col. 5**
Row. 1°	1	3	1	1	1
Row. 2°	3	1	2	2	1
Row. 3°	1	2	1	1	1
Row. 4°	1	2	1	0	4
Row. 5°	1	1	1	4	0

Figure 1.3.

Example 1.4.

In Fig. 1.3 depicts the incidence matrix $M_5^5(G)$ of graph **G** shown in Fig.1.2.

In the case of the investigate of asymmetric non-commutative relationships that are often encountered in real life (for example, the relationship between a great boss and his a subordinates will be, practically, one-sided) convenient to use instead of the ordinary graphs the digraphs in which the edges corresponding to the pairwise relationship have the directions.

The formal definition of the digraph is follows.

Definition 1.3.

A simple directed graph $\mathfrak{I}$, or a simple digraph $\mathfrak{I}$, is called an ordered pair:

$$\mathfrak{I} = (\mathbf{V}(\mathfrak{I}), \mathbf{E}(\mathfrak{I})),\ \mathbf{E}(\mathfrak{I}) \subseteq \mathbf{V}(\mathfrak{I}) \coprod \mathbf{V}(\mathfrak{I}), \tag{1.5}$$

where $\mathbf{V}(\mathfrak{I}) \neq \varnothing$ is a set called the set of **vertices** $\mathfrak{I}$, and $\mathbf{E}(\mathfrak{I})$, which is a subset of the free union of its vertices $\mathbf{V}(\mathfrak{I})$ and is called **arrows** $\mathfrak{I}$. Moreover, the first component $\mathbf{v}_1$ of the arrows $\mathbf{e} = \langle v_1, v_2 \rangle \in \mathbf{E}(\mathfrak{I})$ is called a **tail e**, and the second component $\mathbf{v}_2$ is called a **spike** of edge **e**. In the simple digraph $\mathfrak{I}$, for any two of its vertices $\mathbf{v}_1$ and $\mathbf{v}_2$ there can be no more than one arrow $\mathbf{e} = \langle \mathbf{v}_1, \mathbf{v}_2 \rangle$ or $\mathbf{e}^{\#} = \langle \mathbf{v}_2, \mathbf{v}_1 \rangle$, or none.

Example 1.5.

The simple digraph $\mathfrak{I}$ is depicted in figure 1.4. Its vertices will set

$$\mathbf{V}(\mathfrak{I}) = \{v_1, v_2, v_3, v_4, v_5\},$$

and the elements marked with red circles, and its edges will be the set

$$\mathbf{E}(\mathfrak{I}) = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11}, e_{12}, e_{13}, e_{14}\}.$$

The arrows are marked with blue.

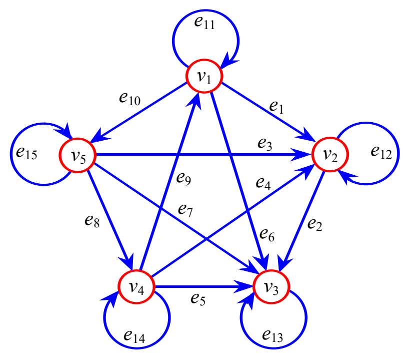


Fig. 1.4.

The digraph $\mathfrak{I}$ very well depicts the work of a magnificent, busy administration. The vertices of $\mathfrak{I}$ will be the members of the administration, and the arrows indicate the transfer of the directive, or instructions one member of the administration to another.

Definition 1.4.

A **digraph G** is an ordered pair

$$\mathbf{G} = (\mathbf{V}(\mathbf{G}), \mathbf{E}(\mathbf{G})), \tag{1.6}$$

where $\mathbf{V}(\mathbf{G})$ is the set of its vertices, and the set $\mathbf{E}(\mathbf{G})$ of it's the arrows can be represented as the union $\mathbf{E}(\mathbf{G}) = \mathbf{E}_1(\mathfrak{I}_1) \cup \ldots \cup \mathbf{E}_n(\mathfrak{I}_n)$ of the set of arcs $\mathbf{E}_i(\mathfrak{I}_i)$, $\mathbf{i} = [\mathbf{1}, \mathbf{n}]$, of some simple digraphs $\mathfrak{I}_i = (\mathbf{V}(\mathbf{G}), \mathbf{E}_i(\mathfrak{I}_i))$, which are identical with **G** the set of its vertices.

Example 1.6.

A digraph **G** is depicted in figure 1.5. The vertices will be set

$$\mathbf{V}(\mathbf{G}) = \{v_1, v_2, v_3, v_4, v_5\},$$

and the set of its the arrows will be set

$$\mathbf{E}(\Delta) = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11}, e_{12}, e_{13}, e_{14}, e_{15}, e_{16}, e_{17}, e_{18}, e_{19}, e_{20}\}.$$

The multiarrows between two different the vertices can be interpreted as more than one number of the instructions, or the directives coming from one member, or organization of society to another, which is most often encountered in real state control.

Let $\mathbf{G} = (\mathbf{V}(\mathbf{G}), \mathbf{E}(\mathbf{G}))$ be a digraph (1.6). If the arc $\mathbf{e} = \langle \mathbf{v}_1, \mathbf{v}_2 \rangle \in \mathbf{E}(\mathbf{G})$, then the vertex $\mathbf{v}_2$ is called an **external** with respect to the vertex $\mathbf{v}_1$, and the vertex $\mathbf{v}_1$ is an **internal** with respect to $\mathbf{v}_2$. At the same time, we call the arc $\mathbf{e} = \langle \mathbf{v}_1, \mathbf{v}_2 \rangle$ such that an **outgoing** from the vertex $\mathbf{v}_1$ and an **entering** the vertex $\mathbf{v}_2$. The two arcs $\mathbf{v}_1$ and $\mathbf{v}_2$ will be called a **parallel** if they have equivalence the tails and the spikes. Denote by $\mathbf{n}_G(\mathbf{V})$ and $\mathbf{n}_G(\mathbf{E})$ the number of vertices and arrows of the digraph **G**, respectively. The number $\mathbf{n}_G(\mathbf{V})$ will be called an **order**, and the number $\mathbf{n}_G(\mathbf{E})$ is a **volume** of the digraph **G**.

Through **fl**(v$_1$) we denote the set of all arrows of the digraph **G** that belong to the vertex **v$_1$**, and through **st**(v$_1$) the set of all arcs **G** that come out of the vertex **v$_1$**. **fl**(v$_1$) is called a **flower** with a center in **v$_1$**, and **st**(v$_1$) is a **star** with a center in **v$_1$**. The arrow in which the tail and the spik coincide is called a **hook**. The vertex **v$_1$** for which is performed **fl**(v$_1$) =∅ (**st**(v$_1$) = ∅, **fl**(v$_1$) = **st**(v$_1$) = ∅), respectively, are called a **root** (a **leaf**, a **bean**) of the graph **G**. A **distiches L$^{\to}$**(v$_1$,v$_{n+1}$) in the digraph **G** that comes out of the vertex **v$_1$** and comes to the vertex **v$_2$** is called the alternating sequence of the form:

$$v_1\, e_1\, v_2\, e_2\, v_3 \ldots e_n\, v_{n+1} \qquad (1.7)$$

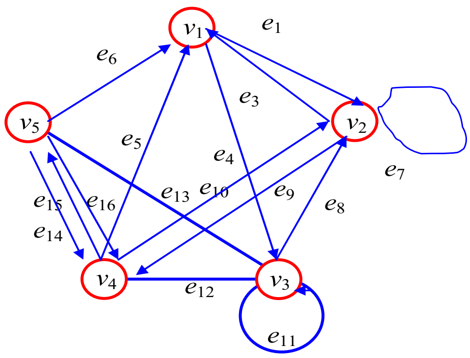


Fig. 1.5.

the vertices and the arrrows, which begins with the vertex v_1 and ends with the vertex **v$_{n+1}$**. In addition, all vertices **v$_i$** with odd indices **i** are external to the arrrows **e$_i$**, and the vertices **v$_i$** with odd indices **j** are internal to the arrrows **e$_j$**. The vertex v_1 is called the **input**, and the vertex **v$_{n+1}$** is the output of the **distiches L$^{\to}$(v_1,v_{n+1}).** A **dipath P$^{\to}$(v_1,v_{n+1})** in the digraph **G** is called the distiches (1.7), in which all the vertices that belong to it, except that it is possible to enter v_1 and output **v$_{n+1}$**, are pairwise different. If the input v_1 and the output **v$_{n+1}$** of the dipath **C$^{\to}$(v_1,v_{n+1})** coincide, then **C$^{\to}$(v_1,v_{n+1})** is called a **dicycle**. The vertex v_{00} is called an **accessible** from the vertex v_0 in the graph **G**, if there exists a dipath **P$^{\to}$(v_0,v_{00})** in **G** with the input in v_0 and output in v_{00}.

Example 1.7.

In figure 1.5 depicts an **G** digraph. The tail of the arrow **e$_1$** will be the vertex **v$_1$**, and the spike of the arrow **e$_1$** will be the vertex **v$_2$**. The arrow **e$_1$** comes from the vertex **v$_1$** and enters the vertex **v$_2$**. The arrows **e$_{10}$** and **e$_{14}$** are parallel, and arrows **e$_3$** and **e$_1$** are not. The arrow **e$_7$** is the hook **n$_G$(V) = 5**, and **n$_G$(E) = 16**. The star in the center **v$_3$** will be the set of edges {**e$_{11}$** , **e$_{13}$**, **e$_{12}$**, **e$_8$**}. The flower in the center **v$_3$** will be a set of edges {**e$_{11}$** , **e$_{13}$**, **e$_{12}$**, **e$_8$**}. There are no the roots, the leaves and the beans in the digraph **G**. The distiches **L$^{\to}$(v$_1$,v$_2$)** will be the sequence **v$_1$ e$_2$ v$_2$ e$_7$ v$_2$**. But **L$^{\to}$(v$_1$,v$_2$)** is not be the distiches. The dipath **P$^{\to}$(v$_1$,v$_3$)** there will be a sequence **v$_1$ e$_1$ v$_2$ e$_9$ v$_4$ e$_{12}$ v$_3$**. But **P$^{\to}$**(v$_1$,v$_3$) das not be the dicycle. The dicycle is the sequence **v$_1$ e$_1$ v$_2$ e$_9$ v$_4$ e$_5$ v$_1$**.

An excellent example of constructing the dicycle in the digraph is the daily route of the postman.

Let **G** = (**V(G)**, **E(G)**) be the digraph. A **coloring of the vertices** of the digraph **G** is the map φ: **V(G)** ⟶ Θ where Θ is marked with a set of its various colors. A **coloring of the arrows** of the digraph **G** is the map ψ: **E(G)** ⟶ Ξ, and Ξ is marked with the set of different colors that painted the arrows. Obviously, the sets Θ and Ξ are called a **colors**. If Θ = ***R***∨**Z**∨***Q*** , in other words, if the names of colors are marked by the real, the integer, or the rational numbers, then the **G** digraph of which has painted the vertices of these numbers is called a **vertex-weighted** digraph. If Ξ = ***R***∨*Z*∨***Q***, then the **G** digraph, which painted the arrows with real, integer, or rational numbers, is called an **arrows-weighted** digraph.

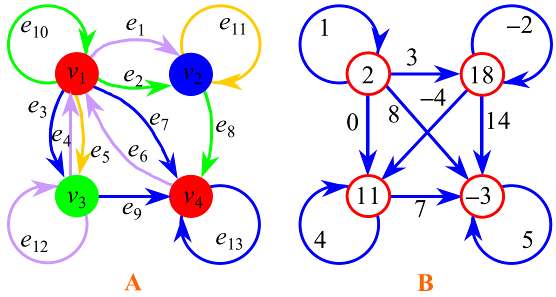


Fig. 1.6.

Example 1.8.

1. Look at figure 1.

2. In figure 1.6.A. depicts the digraph **G** which has painted all the vertices and the arrows.

The set Θ = {red, blue, green}, and the set Ξ = {cumin, blue, green, yellow}.

3. In the figure 1.6.B is depicted is the digraph in which all the vertices and the arrows are colored with the integers, and hence it is vertex-and arrows-weighted. He has the set Θ = {2, 18, -3, 11}, and the set Ξ = {1, 3, -2, 0, 8, -4, 14, 4, 7, 5}.

2. Ordered of socium data collection

Let α is a person and ℜ is a socium.

Definitions 2.1.

A binary relation

$$\mathbf{x}_{\Re}\succ_{\alpha}\mathbf{y} \tag{2.1}$$

which symbolizes the statement that the α persons' prefers according to the indication of the socium ℜ the variant **x** to the variant **y**. A binary relation $\mathbf{x}_{\Re}\succ_{\alpha}\mathbf{y}$ may be called a **preference relation** of the person α to the indication of the socium **ℜ.**

A binary relation

$$\mathbf{x}_{\Re}\nabla_{\alpha}\mathbf{y} = \{(\neg(\mathbf{x}_{\Re}\succ_{\alpha}\mathbf{y})) \wedge (\neg(\mathbf{y}_{\Re}\succ_{\alpha}\mathbf{x}))\} \tag{2.2}$$

which symbolizes the statement that the α persons' according to the indication of the socium ℜ is indifferent between the variant **x** to the variant **y** and the variant **y** to the variant **x**. A binary relation $\mathbf{x}_{\Re}\nabla_{\alpha}\mathbf{y}$ may be called an **indifference relation** of the person α to the indication of the socium **ℜ.**

A binary relation

$$\mathbf{x}\ {}_{\Re}\rightrightarrows_{\alpha}\mathbf{y} = \{((\mathbf{x}_{\Re}\succ_{\alpha}\mathbf{y})) \vee ((\mathbf{x}_{\Re}\nabla_{\alpha}\mathbf{y}))\} \tag{2.3}$$

which symbolizes the statement that the α person according to the indication of the socium ℜ prefers the variant **x** to variant **y** or is indifferent between the variant **x** to the variant **y** and the variant **y** to the variant **x**. We may call this binary relation $\mathbf{x}_{\Re}\rightrightarrows_{\alpha}\mathbf{y}$ may be called a **strongly preference relation** of the α to the indication of the socium **ℜ.**

First, we ask socium axiomatics.

The axioms 1-6 see in [9].

Axiom 7 areflexivity of preference to the indication of socium.

$$\forall(\mathbf{x})\forall\alpha \Rightarrow [\neg(\mathbf{x}_{\Re}\succ_{\alpha}\mathbf{x})\}] \tag{2.4}$$

For any variant **(x)** and any person α to the indication of a socium ℜ $(\mathbf{x}_{\Re}\prec_{\alpha}\mathbf{x})$ does not hold.

Axiom 8 transitivity of preference to the indication of socium.

$$\forall(\mathbf{x,y,z})\forall\alpha \Rightarrow [\{(\mathbf{x}_{\Re}\succ_{\alpha}\mathbf{y})\wedge(\mathbf{y}_{\Re}\succ_{\alpha}\mathbf{z})) \Rightarrow \neg(\mathbf{x}_{\Re}\succ_{\alpha}\mathbf{z})\}] \tag{2.5}$$

For any triple of variants **(x,y,z)** and any person α to the indication of a socium ℜ, if $(\mathbf{x}_{\Re}\succ_{\alpha}\mathbf{y})$ and $(\mathbf{y}_{\Re}\succ_{\alpha}\mathbf{z})$ hold, then $(\mathbf{x}_{\Re}\succ_{\alpha}\mathbf{z})$ holds.

Axiom 9 transitivity of indifference to the indication of socium.

$$\forall(\mathbf{x,y,z})\forall\alpha \Rightarrow [\{(\mathbf{x}_{\Re}\nabla_{\alpha}\mathbf{y})\wedge(\mathbf{y}_{\Re}\nabla_{\alpha}\mathbf{z})) \Rightarrow (\mathbf{x}_{\Re}\nabla_{\alpha}\mathbf{z})\}] \tag{2.6}$$

For any triple of variants **(x,y,z)** and any person α to the indication of a socium ℜ, if $(\mathbf{x}_{\Re}\nabla_{\alpha}\mathbf{y})$ and $(\mathbf{y}_{\Re}\nabla_{\alpha}\mathbf{z})$ hold, then $(\mathbf{x}_{\Re}\nabla_{\alpha}\mathbf{z})$ holds.

Axiom 10 connectedness of strongly preference to the indication of socium.

$$\forall(\mathbf{x,y})\forall\alpha \Rightarrow [\{(\mathbf{x}_{\Re}\rightrightarrows_{\alpha}\mathbf{y})\vee(\mathbf{y}_{\Re}\rightrightarrows_{\alpha}\mathbf{z}))\ \}] \tag{2.7}$$

For any pair of variants **(x,y)** and any person α to the indication of a socium ℜ, if $(\mathbf{x}\rightrightarrows_{\alpha}\mathbf{y})$ or $(\mathbf{y}\rightrightarrows_{\alpha}\mathbf{z})$ holds.

Axiom 11 transitivity of strongly preference to the indication of socium.

$$\forall(\mathbf{x,y,z})\forall\alpha \Rightarrow [\{(\mathbf{x}_{\mathfrak{R}}\Rrightarrow_{\alpha}\mathbf{y})\wedge(\mathbf{y}_{\mathfrak{R}}\Rrightarrow_{\alpha}\mathbf{z})) \Rightarrow (\mathbf{x}_{\mathfrak{R}}\Rrightarrow_{\alpha}\mathbf{z})\}] \quad (2.8)$$

For any triple of variants **(x,y,z)** and any person α to the indication of a socium $\mathfrak{R}$, if $(\mathbf{x}_{\mathfrak{R}}\Rrightarrow_{\alpha}\mathbf{y})$ and $(\mathbf{y}_{\mathfrak{R}}\Rrightarrow_{\alpha}\mathbf{z})$ hold, then $(\mathbf{x}_{\mathfrak{R}}\Rrightarrow_{\alpha}\mathbf{z})$ holds.

3. Ordered of socium on linear graph data collection

Let α is a person, $\mathfrak{R}$ is a socium, and the distiches $\mathbf{L}^{\rightarrow}(x,y, \ldots,w)$, see figure 3.1.

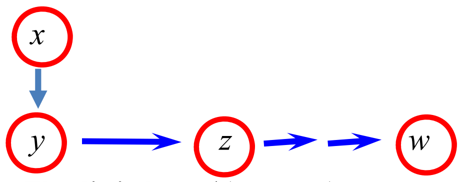


Fig. 3.1. The distitches L→(*x*,y, …,*w*).

Definitions 3.1.

An ordered of socium $\mathfrak{R}$ on linear graph the distiches $\mathbf{L}^{\rightarrow}(x,y, \ldots,w)$ is defined:

1) when α need to make a lot (≥ 2) of decisions,
2) as an ordering defined on an issue: what is to be chosen first, what second, third and so on.

In other words, all variants are somehow ordered by the society according to is decision **rule** $\mathfrak{R}$. Rule will be the digraphs $\mathbf{L}^{\rightarrow}(x,y, \ldots,w)$, see figure 3.1.

A binary relation

$$\mathbf{x}_{\mathfrak{R}}\succ_{\alpha} \mathbf{y}_{\mathfrak{R}}\succ_{\alpha}\ldots\ \mathbf{P}^{\rightarrow}(x,y, \ldots,w)\ldots_{\mathfrak{R}}\succ_{\alpha}\mathbf{w} \quad (3.1)$$

signifies the α persons' prefers according to the indication of a socium $\mathfrak{R}$, according to is decision rule by the graphs $\mathbf{L}^{\rightarrow}(x,y, \ldots,w)$ the variant **x** to the variant **y**; furzer the variant **y** to the variant **z**; and third so on to finite distant the vertex **w** of the distiches $\mathbf{L}^{\rightarrow}(x,y, \ldots,w)$, see figure 3.1.

A binary relation

$$\mathbf{x}_{\mathfrak{R}}\nabla_{\alpha}\mathbf{y}\ldots\ \mathbf{L}^{\rightarrow}(x,y, \ldots,w)\ldots_{\mathfrak{R}}\nabla_{\alpha}\mathbf{w} = \{(\neg(\mathbf{x}_{\mathfrak{R}}\succ_{\alpha}\mathbf{y})) \wedge (\neg(\mathbf{y}_{\mathfrak{R}}\succ_{\alpha}\mathbf{x}))\ldots \neg\,(\cdot_{\mathfrak{R}}\succ_{\alpha}\mathbf{w})) \wedge (\neg(\mathbf{w}_{\mathfrak{R}}\succ_{\alpha}\cdot))\} \quad (3.2)$$

signifies the α persons' prefers or indifference preference relation according to the indication of a socium $\mathfrak{R}$, according to is decision rule by the graphs $\mathbf{L}^{\rightarrow}(x,y, \ldots,w)$ the variant **x** to the variant **y**; furze the variant **y** to the variant **z**; and third so on to finite distant the vertex **w** of the distitches $\mathbf{L}^{\rightarrow}(x,y, \ldots,w)$, see figure 3.1.

A binary relation

$$\mathbf{x}_{\mathfrak{R}}\Rrightarrow_{\alpha}\mathbf{y}\ldots\ \mathbf{L}^{\rightarrow}(x,y, \ldots,w)\ \ldots\ _{\mathfrak{R}}\Rrightarrow_{\alpha}\mathbf{w} = \{(\mathbf{x}_{\mathfrak{R}}\succ_{\alpha}\mathbf{y}) \vee (\mathbf{x}_{\mathfrak{R}}\nabla_{\alpha}\mathbf{y})\ \ldots(\cdot_{\mathfrak{R}}\succ_{\alpha}\mathbf{w}) \vee (\cdot_{\mathfrak{R}}\nabla_{\alpha}\mathbf{w})\} \quad (3.3)$$

signifies the α persons' indifference preference relation according to the indication of a socium $\mathfrak{R}$, according to is decision rule by the graphs $\mathbf{L}^{\rightarrow}(x,y, \ldots,w)$ the variant **x** to the variant **y**; furze the variant **y** to the variant **z**; and third so on to finite distant the vertex **w** of the distitches $\mathbf{L}^{\rightarrow}(x,y, \ldots,w)$, see figure 3.1.

4. Case of a general graph

Let α is a person, $\mathfrak{R}$ is a socium, and a general graph Γ, see figure 4.1.

Example 4.1.

1. In Figure 4.1. depicts a digraph

$$\Gamma = (\mathbf{V}(\Gamma), \mathbf{E}(\Gamma)) \quad (4.1)$$

which has painted all the vertices and arcs. Set **V(Γ)** = {red, blue, green} *and set* **E(Γ)** = {lila, cumin, blue, green, yellow}.

2. Red vertices (**x, z**) *are a social solutions, green vertices* (**w**) *– an ecological solutions, blue vertices* (**y**) *– a financial solutions.*

3. Edges blue –($_{\mathfrak{R}}\succ_{\alpha}$), green –($_{\mathfrak{R}}\nabla_{\alpha}$), yellow-($_{\mathfrak{R}}\Rrightarrow_{\alpha}$), lila-($_{\mathfrak{R}}\Rrightarrow_{\alpha}$), cumin -–($_{\mathfrak{R}}\nabla_{\alpha}$).

4.The Social 𝔑 initially proposed from the social service **x** *to apply strictly* ($\Re\exists_\alpha$) *to the financial service* (**y**), *then to remind* ($\Re\nabla_\alpha$)) *from the social service* **x** *financially* **y**, *and so on, by the graph* **Γ, …,** *finally, the social service* (**z**) *solves some of its questions* (**e**$_{13}$).

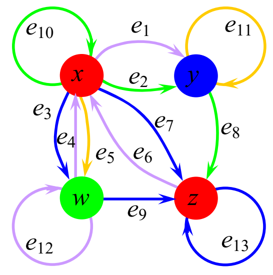


Fig. 4.1.

5. Conclusion

As Example 4.1 shows with our methods from the first three articles [8]-[9], we can easily simulate any real sociometric problem for one person. And since all our models are reduced to computational tasks over finite graphs, then we will be able to construct (or have already constructed, see [4], [6]-[7]) good algorithms for their solution.

Ahead of the mathematical theories for multiple persons and multiple alternatives.

Pict. 2. Grosh. The Socium.

References:

1. *Gritsak V.V.* Logic and Categorical Theory of Natural Science. — Kyiv: SVITOZIR-ACADEMIA, 1995. — 322 p.
2. *Gritsak V.V., Michalevich V.S.* Mathematical Theory of Democracy. — Moscow: Progress, 1984. — 230 p.
3. *Gritsak-Groener V.V., Gritsak-Groener J.* Computation of Alternative Conditions for Social Orderings Optimal Democratic Decision. // Физика сознания и жизни, космология и астрофизика. — 2014. — № 2.
4. *Gritsak-Groener V.V.* Theory of State. Choice. — Kyiv-München: SVITOZIR-ACADEMIA, 2000. — 346 p.
5. *Gritsak-Groener V.V., Gritsak-Groener J.* Global Controls and Sufficient Conditions of Goduniquely // Соционика, ментология и психология личности. — 2012. — №1.
6. *Gritsak-Groener V.V.* The Mathematical Theory of the State. Construction of the State System. — Kyiv-München: SVITOZIR-ACADEMIA, 2003.
7. *Gritsak-Groener V.V.* The Mathematical Theory of the State Management, v. 2. — Kyiv-München: SVITOZIR-ACADEMIA, 2005.
8. *Gritsak-Groener V.V., Gritsak-Groener J.* Classical mathematical sociometry II. Mathematics, poetry, picture. // Физика сознания и жизни, космология и астрофизика. — 2018. — № 3-4.
9. *Gritsak-Groener V.V., Gritsak-Groener J.* Classical mathematical sociometry I. // Физика сознания и жизни, космология и астрофизика. — 2018. — № 1-2.

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Гритсак-Грёнер В.В., Гритсак-Грёнер Ю.

Классическая математическая социометрия. Часть III.
Графические методы упорядочивания социума

В статье развиваются математические методы принятия социальных решений, когда упорядоченность решений задается произвольным конечным графом. Например: упорядоченные, взвешенные и раскрашенные множества. Постановка проблемы никогда не рассматривалась в математической социометрии. Разработанные в наших статьях математические методы решения прямых и двойственных математических задач задач единого исполнителя под управлением социума **α** по схеме произвольного конечного графа **G** позволяют алгоритмически решать любую реальную задачу социометрического планирования.

Ключевые слова: социум, мульти диграфы, цвет, порядок.