SYNERGETICS AND THEORY OF CHAOS

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COMPUTATION OF ALTERNATIVE CONDITIONS FOR SOCIAL ORDERINGS OPTIMAL DEMOCRATIC DECISION

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In this paper we study the stationary damped transitive processes. The main application area is the structure of social relations. A society, regarded as a category, according to certain rules is that characterize the population of citizens with the same interests or political preferences. This article was intended as to motivate of decision making for conditions for social orderings. We can be deduced by recognizing that a best social orderings is democracy. The definition of democracy is one of the most controversial matters in the field of the sciences. So that even an introduction to the topic could form the subject of our articles. Also we study algorithms of alternative conditions for social orderings.

Key words: category, object, morphism, democracy.

Чтобы вырвать век из плена, Чтобы новый мир начать, Узловатых дней колена Нужно флейтою связать. Это век волну колышет Человеческой тоской, И в траве гадюка дышит Мерой века золотой.

Осип Мандельштам

Реферат

В случае управления относительно небольшими коллективами социум S можно интерпретировать конечными графами и категориями. Существуют примеры (см. расчёт по нашему алгоритму), когда управляющий центр состоит не из одного элемента (в нашем примере из двух — лидер и нелидер, но имеющий большее число связей в социуме S, например — секретарь или первый помощник лидера).

Параграфы 2—4 посвящены построению алгоритма расчета расчёта потенциалов (силы!) членов социума, заданного конечной категорией **К**, что со-ответствует, например, партийной организации (в действительности, наш при-мер взят из реального партийного объединения) и наш алгоритм может быть использован для партийного управления строительства реальной партийной или другой политической организации. Неожиданностью, при расчёте нашего примера, оказалось наличие члена S обладающего более значительным потенциа-лом чем лидер. (В реальности, это сказалось в катастрофе для реальной S).

Параграфы 5-6 содержат первую теорему о наличии единого центра, смо-три нашу теорему о наличии единого управляющего центра. Теорема доказана при довольно слабых естественных ограничениях (возможность персчета членов S (вложение в $\mathbf{R}^{\mathbf{n}}$), выпуклость, линейность, которые всегда могут быть получе-ны при соответсвующей интерпретации данных и тому подобное). Существен-ным ограничением есть отсуствие ограничений конечности. Это реальный прин-цип, доказательство, которого, пока не понятно как доказывать, созможно и во-

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обще невозможно в рамках математики. Замечу, что при построении реальной модели глобального управления социума S, нам необходимо будет достраивать категорию K до топоса T,

а как показала **Ольга Карпенко** [7], не существует конечных топосов T_{κ} (одновременно с конечным числом объектов и морфизмов), что привело её научного руководителя и одного из авторов статьи к догадке о **бесконечности**, **единственности** и **недостижимости** глобального центра управления. Малая теорема подтверждает, лишь первые два условия. Для недостижимости объект управления недостаточно «глобальный». Да и для формулировки основной теоремы нам необходимо построить теории глобальности и недостижимости, что мы и сделаем в последующих наших статьях.

Заметим, что большинство научных работ, затрагивающих гуманитарную сферу переписаны из работ своих великих предшественников или с потолка. Поэтому, не содержат никаких доказательств. Поставив перед собой такие важные и глобальные цели, мы обязаны проводить точные и единственно возможные убедительные математические доказательства.

1. Introduction

This article was intended as to motivate of decision making for conditions for social orderings. We can be deduced by recognizing that a best social orderings is *democracy*. The definition of democracy is one of the most controversial matters in the field of the sciences. So that even an introduction to the topic could form the subject of our articles.

For the first time the representation of society as a category

$$K = (Ob(K), Mor(K))$$
 (1)

has been proposed in [4]. In this model, objects $\mathbf{Ob}(K)$ in the category K are the people such that the citizens of this socium S [5] (see figure 1), and morphisms $\mathbf{Mor}(K)$ are an inter relationship between them.

Work in this direction was continued in [1]-[2], [4]-[6]. This paper is a continuation and development of research [3].

Object of study is a mathematical model describing the stationary, damped, transitive and reflexive processes. Its scope in this paper is the structure of socium relations. However, our theory can be successfully applied also to study the sparse physical, chemical, biological and other processes after a slight change in terminology.

The proposed model adequately describes the socium processes. Indeed, the existence of man (the object $o_1 \in Ob(K)$ of our category K) is determined by its relationships (morphisms Mor(K)) with other people, God, the ourselves. The types of socium struc-



Fig. 1. Grosz. The Socium.

tures are economic, ethnographic, and others. These relationships Mor(K) may be single-type or two-

type. For two relations between the three successive objects $o_1, o_2, o_3 \in Ob(K)$ there is always a relationship between extreme objects, which is a consequence of relations with the central object of the extreme (see in Fig. 2). This ratio can be interpreted as the product of relations and their transitivity. The relations Mor(K) "by someone" weaker with each iteration, so that such processes are damped.

Further, each person has the relationship with oneself, which is reflected in his own thoughts and decisions. This guarantees the existence of the identity morphism $id(o) \in Mor(K)$ for each object $o \in Ob(K)$ (reflexivity). Morphisms id(o) in figure 2 show the symbol of the id and are represented as loops.

Moreover, a **stationary** of our model is its behavior S in a fixed time.

For the same reason, let Mor(K) be a completeness (see [3], [4]).

Finally, an **associatively** is natural property of a relations. The property an associatively means that the socium S is homogeneous. In other words, a preferred classes absent in a socium S.

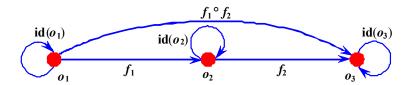


Fig. 2.

We understand: our articles the beginning of research in this direction.

On the other hand, in the public relations elements of the study population are people, citizens of the society along with his individual psychology, the history of their lives and the lives of its people, national and ethnographic features, level of education and other individual parameters. All this should be taken into account in an exact model of social relations. Unfortunately, all these features into account, together with the modern development of computer technology is impossible. But to formalize all of these and other features is not easy.

2. Exact Replica and New Definition

We a fixed set A with elements α , β , γ , ...

Definitions 1. A binary relation on **A** is called

- 1. reflexive if $\alpha \ge \alpha$,
- 2. irreflexive if $\alpha \ge \alpha$,
- 3. complete if $\alpha \geq \beta$ or $\beta \geq \alpha$ holds true for any $(\alpha,\beta) \in A \times A$,
- 4. transitive if $\alpha \geq \beta$, $\beta \geq \gamma$ implies $\alpha \geq \gamma$ (α , β , $\gamma \in A$).

Definitions 2. A binary relation \geq on **A** is called a **preference** if \geq reflexive, transitive, and complete.

Definitions 3. Let a binary relation \geq on **A**. Then

a. $\geq \star = \{(\alpha, \beta) : (\beta, \alpha) \in \geq \}$ is called a **dual** relation of \geq ,

b. $\succ = \geqslant \cap \not \geqslant \star$,

 $c. \approx = \geqslant \cap \geqslant \star.$

We need in a category K = (Ob(K), Mor(K)) and a covariant functor (or functor) $\Phi : K_1 = K_2$, where K_1 , K_2 are a categories.

Definitions 4. Let $K = (\mathbf{Ob}(K), \mathbf{Mor}(K))$ is finite category. An object $\mathbf{o} \in \mathbf{Ob}(K)$ with respect to object $\mathbf{u} \in \mathbf{Ob}(K)$ is called:

- 1. K-equivalent if $Mor(o, u) \neq \emptyset$ and $Mor(u, o) \neq \emptyset$ ($o\Theta_{K}u$);
- 2. large if Mor(o, u) $\neq \emptyset$ (o \triangleright u);
- 3. strictly large if Mor(o, u) $\neq \emptyset$ and Mor(u, o) = \emptyset ($o \triangleright u$);
- 4. much more if $\exists ! \varphi \in Mor(o, u)$ and $Mor(o, u) = \emptyset$ (o > u).

For every fixed $o \in Ob(K)$ let

- 1) $E(K, o) = EK(o) = \{x \in Ob(K) : x\Theta Ko\},\$
- 2) $G(K, o) = GK(o) = \{x \in Ob(K) : x \triangleright o\},\$
- 3) $G \triangleright (K, 0) = G \triangleright K(0) = \{x \in Ob(K) : x \triangleright 0\},\$
- 4) $F(K, o) = FK(o) = \{ x \in Ob(K) : o \triangleright x \},$
- 5) $F \triangleright (K, o) = F \triangleright K(o) = \{ x \in Ob(K) : o \triangleright x \}.$

Now we introduce the following definition.

Definitions 5. The category K is called **weighted** if there exists the map

$$\mathbf{W}: \mathbf{Mor}(\mathbf{K}) \longrightarrow \mathbf{R},\tag{2}$$

where R is the field of real numbers. The number $W(\phi)$ is called the **weight** of the morphism ϕ and the mapping (2) is a **weighting**. We need be to

$$\mathbf{W}(\mathbf{\phi}), \mathbf{W}(\mathbf{\gamma}) \ge \mathbf{W}(\mathbf{\phi} \circ \mathbf{\gamma}) \tag{3}$$

if $f \circ g \in Mor(K)$.

We interpret the rule (3) as a reduction in force under the influence of indirect contact.

Definitions 6. A potential object $o \in Ob(K)$ is the sum

$$\sum_{x \xrightarrow{\phi_i} o} W(\phi_i) - \sum_{o \xrightarrow{\psi_j} y} (\psi_j)$$
(4)

all weights of morphisms Mor(K) incoming to o and the sum all weights of morphisms Mor(K) outgoing to o, see (4).

A potential describes the overall impact of a person o to other people in socium S. A positive value of potential describes the influential person \mathbf{o} , and a negative value of potential describes prone a person \mathbf{o} to influence by other people.

Let a peoples socium S is the category $K = (\mathbf{Ob}(K), \mathbf{Mor}(K))$ such that $\mathbf{Ob}(K) \subset \mathbb{R}^n$. Moreover in this article we shall assume that $\mathbf{Ob}(K)$ is a **linear space**, i. e. consequently operations $\mathbf{o} - \mathbf{u}$, $\mathbf{o} + \mathbf{u}$, **ho** are defined if $\mathbf{o}, \mathbf{u} \in \mathbf{Ob}(K)$, $\mathbf{h} \in \mathbb{R}$, and be fulfilled the usual algebraic laws.

Definitions 7. A set $A \subseteq Ob(K)$ is called **convex** if $x^0, x^1 \in A$ implies

$$xh = hx1 + (1-h) x0 \in A (0 \le h \le 1).$$
 (5)

Let $K_A = (A \text{ , Mor}(A))$ is subcategory, $A \subseteq Ob(K)$ is convex, where Mor(A) is a set of morphisms $\phi : o_1 \longrightarrow o_2$ such that $o_1, o_2 \in A$, $\phi \in Mor(K)$. Then K_A is called

- 1. weakly convex if $\gamma : x^0 \longrightarrow x^1$ implies $\eta : x^h \longrightarrow x^1$ such that $\gamma, \eta \in Mor(A)$ (0 < h < 1),
- 2. convex if $x^0 \triangleright x^1$ implies $x^h \triangleright x^1$ (0 < h < 1),
- 3. **strongly** convex if $\gamma : x^0 \longrightarrow x^1$, $x^0 \neq x^1$ implies $x^h \triangleright x^1$ (0 < h < 1). Similarly, strongly convex implies convexity but this in turn does not imply weak convexity.

Definitions 8. A category $K = (\mathbf{Ob}(K), \mathbf{Mor}(K))$ is called **topologized** if, for all $o \in \mathbf{Ob}(K)$,

 $\mathcal{G}^{^{\triangleright}}_{\kappa}(o)$ and $\mathcal{F}^{^{\triangleright}}_{\kappa}(o)$ are closed sets, $\mathcal{G}_{\kappa}(o)$ and $\mathcal{F}_{\kappa}(o)$ are open sets in a topology T on $Ob(\kappa)$.

3. Algorithm of Computation for Forces in Socium

Let the category S is weighted, and function

$$W: Mor(K) \longrightarrow \square, \tag{6}$$

see (2)–(4).

Listing procedures of algorithm of computation for forces in socium S in pseudocode is as follows:

Algorithm 1

```
Input: morphisms Mor(1: m,1:2) of category K, weights of directly influence W(1: m), and factors of
weakening of influence alpha(1: n).
A=I(m); {Commentary 1}
for k=1:m {Commentary 2}
 hv=Mor(k, 1); {Commentary 3}
 pr \leftarrow (номера всех Mor(:,2)=hv); { Commentary 4}
 for k1=1:length(pr) { Commentary 5}
  A(k,k1) = -alpha(hv); {Commentary 6}
 end {for k1}
end {for k}
Weight \leftarrow (Commentary 7)
Pot=0; {Commentary 8}
for k=1:n {Commentary 9}
 hv \leftarrow \{(Mor(:, l) = k); \{commentary 10\}\}
 vi \leftarrow \{(Mor(:,2)=k); \{Commentary 11\}\}
 Pot(k)=sum(Weight(hv))-sum(Weight(vi)); { Commentary 12}
end {for k}
Output: Weight(1:m) – weights of morphisms of category Mor(1: m,1:2);
 Pot(1:n) — potentials of objects of category K.
```

Example

The results of this algorithm is in figure 4, figure 5. The category κ (the socium S) demonstration in figure 3 here shows immediate morphisms without loops. In this example, we assign by immediate morphisms of κ weight is equal 0.5, see fig.3. The algorithm is computation of the function κ (6).

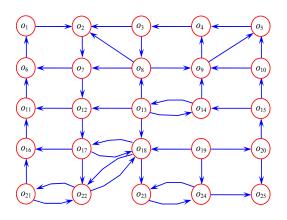


Fig. 3.

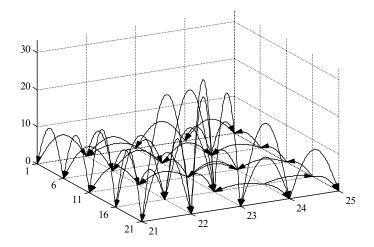


Fig. 4.

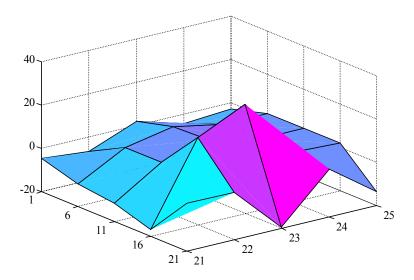


Fig. 5.

The highest potential was not a leader (object v19), and in v18, where he is 30.55275. At the same time, the potential leader of our model is only 3 — ten times less. Morphisms have the greatest weight associated with the object v18. This result explains why sometimes in public relations head of the office (or the secretariat, administration, etc.) is more influential than a leader, he has access and influence on many people. The obtained values of weights are approximate morphisms. Algorithms for calculating the exact weight and developed by us in the framework of measurement theory of social relations.

4. Base Lemmas

Suppose $K = (\mathbf{Ob}(K), \mathbf{Mor}(K))$ is category such that $\mathbf{Ob}(K)$ $\subset \mathbb{R}^n$. Moreover in this article we shall assume that $\mathbf{Ob}(K)$ is a linear space. A set $\mathbf{A} \subseteq \mathbf{Ob}(K)$ is convex if $\mathbf{x}^0, \mathbf{x}^1 \in \mathbf{A}$ implies

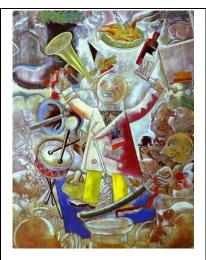


Fig. 6. Grosz. The Influential.

$$x^h = hx^1 + (1-h) x^0 \in A (0 \le h \le 1).$$

We are bound to be proving next base lemmas.

Base Lemma 1. Let the category $K = (\mathbf{Ob}(K), \mathbf{Mor}(K))$ such that $\mathbf{Ob}(K)$ is a linear convex space, $\mathbf{o} \in \mathbf{Ob}(K)$, and $G(K, \mathbf{o}) = \{x \in \mathbf{Ob}(K) : x \triangleright \mathbf{o}\}$. Then $\mathbf{Ob}(K)$ is weakly convex iff $G(K, \mathbf{o})$ is convex for all $\mathbf{x} \in \mathbf{Ob}(K)$.

Proof. Let x^0 , $x^1 \in \mathcal{G}_K(o)$ for some $o \in Ob(K)$. Since Mor(K) is completeness, follows that $x \in A \setminus A \setminus A$

and then $x^h \triangleright x^1 \triangleright o \ (0 \le h \le 1)$. By transitivity of multiplication of morphisms $Mor(K) \ x^h \triangleright o$, that is, $x^h \in \mathcal{G}(K,o)$ and so $\mathcal{G}(K,o)$ is convex.

Inversely, assume $\mathcal{G}(K, \mathbf{0})$ is convex for all $\mathbf{x} \in \mathbf{Ob}(K)$. If $\mathbf{x}^1 \triangleright \mathbf{x}^0$, then $\mathbf{x}^1, \mathbf{x}^0 \in \mathcal{G}(K, \mathbf{x}^0)$, consequently $\mathbf{x}^h \in \mathcal{G}(K, \mathbf{x}^0)$, $\mathbf{x}^h \triangleright \mathbf{x}^0$ and " \triangleright " is welly convex.

Base Lemma 2. Let the category K = (Ob(K), Mor(K)) such that Ob(K) is a linear convex space, $o \in Ob(K)$, and

$$G(K, 0) = \{x \in Ob(K) : x \triangleright 0\}, E(K, 0) = \{x \in Ob(K) : x \cap 0\}.$$

Then Ob(K) is strongly convex iff G(K, 0) is convex for all $x \in Ob(K)$ and $\mathcal{E}_K(0)$ does not contain a line segment.

Proof. Let $\mathbf{Ob}(K)$ is strongly convex. If $\mathbf{x}^1, \mathbf{x}^0 \in \mathcal{G}(K, \mathbf{x}^*)$ for some $\mathbf{x}^* \in \mathbf{Ob}(K)$, then

$$\mathbf{x0} \stackrel{J}{=} \mathbf{x1} \triangleright \mathbf{x}^*$$

where $f \in Mor(K)$. Thence

$$xh \triangleright x1 \triangleright x^*$$

so $\mathbf{x}^h \in \mathcal{G}(K, \mathbf{x}^*)$. Consequently $\mathbf{Ob}(K)$ is strongly convex. Moreover, for $\mathbf{x}^1 \neq \mathbf{x}^0$ and \mathbf{x}^1 , $\mathbf{x}^0 \in \mathcal{E}_K(\mathbf{x}^*)$,

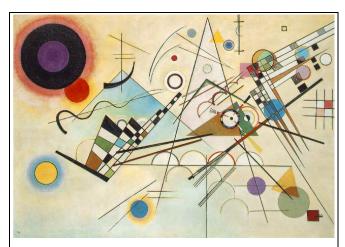


Fig. 7. Wasyl Kandynskij. The preparation to mathematical proof.

x0 OK x1 OK xh.

Thence

$$xh \triangleright x1 \Theta K x^*, xh \triangleright x^*, xh \notin EK(x^*).$$

Consequently $\mathcal{L}_{\kappa}(\mathbf{x}^*)$ does not contain a line segment.

Conversely. Let $\mathbf{x}^1 \neq \mathbf{x}^0$ and $\mathbf{x}^1 - \mathbf{x}^0$, where $\mathbf{f} \in \mathbf{Mor}(K)$.

If $\mathbf{x}^1 \triangleright \mathbf{x}^0$ then $\mathbf{x}^0 \triangleright \mathbf{x}^h$ cannot occurs as this would imply

$$x1 \triangleright x0 \triangleright xh \Rightarrow x1, x0 \in G(K, xh) \Rightarrow xh \in G(K, xh),$$

a contradiction, hence $\mathbf{x}^h \triangleright \mathbf{x}^0$. Further, $\mathbf{x}^1 \Theta_K \mathbf{x}^0$ cannot occurs. Let this was true. Then consider \mathbf{x}^η , $0 < \eta \leq h < 1$, and a full line-segment is contained in $\mathcal{E}_K(\mathbf{x}^0)$. Consequently, the only remaining possibility

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is $x^h \triangleright x^0$

If $x^1 \Theta_K x^0$ then $x^1 \Theta_K x^0 \triangleright x^h$ cannot occur for we would have

$$x1, x0 \in G(K, xh), xh \in G(K, xh)$$

which is impossible. Thence

$$x1 \Theta K x0 \square x$$
.

As $\mathcal{E}_{\kappa}(x^0)$ does not contain line-segment, exist is γ , $0 < \gamma < 1$, such that

$$xy \triangleright x0$$
.

Consider x^{η} , $0 < \eta < \gamma$. If $x^{\eta} \Theta_{\kappa} x^{0}$ then there is κ , $0 < \kappa < \eta$, such that

Thence

$$x\gamma, x\kappa \in G(K, x0), x\eta \in G(K, x0).$$

So

$$x\eta \triangleright x0$$

must hold true $0 < \eta < \gamma$ and the same argument works for all η with $\gamma < \eta < 1$.

5. Small Uniqueness Theorem for Steering Body

Theorem (small uniqueness theorem for steering body). Suppose
$$K = (Ob(K), Mor(K))$$
 (7)

is topologized category such that $Ob(K) \subset \mathbb{R}^n$, $Ob(K) \subset \mathbb{R}^n$ is a linear space and compact. Moreover this paragraph Mor(K) is assumed to be endowed strongly convex. Then there is exactly one optimal object $a_0 \in Ob(K)$.

Proof. Let

$$\mathbf{Ob}(\mathbf{K}) \subseteq \bigcup_{o \in Ob(K)} \mathbf{F}(\mathbf{K}, \mathbf{o}), \tag{8}$$

Then we have defined an open covering of **Ob(K)**. So there are

$$x1, ..., xn \in Ob(K)$$

such that

$$\mathbf{Ob}(\mathbf{K}) \subseteq \bigcup_{i=1}^{n} \mathbf{F}(\mathbf{K}, \mathbf{xi}). \tag{9}$$

We may expect

$$\mathbf{x}_1 - \frac{f_2^1}{f_2^2} - \mathbf{x}_2 - \frac{f_3^2}{f_3^2} - \dots - \frac{f_n^{n-1}}{f_n^n} - \mathbf{x}_n$$

where $f_{k+1}^k \in \mathbf{Mor}(\mathbf{K})$. Here (9) and base lemmas 1, 2 occurs that for $\forall \mathbf{x} \in \mathbf{Ob}(\mathbf{K})$ there is an $\mathbf{i} \in [1,n]$ satisfying

$$\mathbf{x}_1 - \frac{f_i^1}{f_i^1} - \mathbf{x}_i \triangleright \mathbf{x}, f_i^1 \in \mathbf{Mor}(\mathbf{K}).$$

In particular, for some i,

$$\mathbf{x}_1 - \frac{f_i^1}{f_i^1} - \mathbf{x}_i \triangleright \mathbf{x}_1, f_i^1 \in \mathbf{Mor}(\mathbf{K}),$$

thence the assumption (8) is false. Then be realized

Ob(K)
$$\square \bigcup_{o \in Ob(K)} F(K, o)$$
.

In another way, there is $a_0 \in Ob(K)$ such that

 $a0 \notin F(K, o), o \in Ob(K)$.

Finally

$$a0 - f - o, o \in Ob(K), f \in Mor(K).$$

Indeed, there exists at least one optimal element of Ob(K). Let x^0 , x^1 both were optimal, then

xh > x0, 0 < h < 1,

would yield a contradiction.

Definitions 9. The optimal object $\mathbf{a_0} \in \mathbf{Ob}(\mathbf{K})$ category \mathbf{K} (7) (a socium S!) is called a **center** C of **steering body** B, where B is a topological neighborhood of point C.

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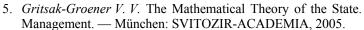




Fig. 8. Ivan Nevidomyj. Body of Control's Center.

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Вычисление альтернативных условий социального порядка – оптимального демократического процесса принятия решений

Исследуются стационарные затухающие транзитивные процессы. Основная область применения — структура общественных отношений, в рамках которой приводятся все примеры. Но после незначительного изменения терминологии наша теория может успешно применяться для исследования физических, химических, биологических и других процессов. Общество, рассматриваемое как категория, по определённым правилам разбивается на части. Все проведённые исследования конструктивны и алгоритмичны. Поэтому для вычисления всех характеристик и параметров нашей модели мы предлагаем алгоритмы, доведенные нами до реально действующих компьютерных программ.

Ключевые слова: категория, объект, морфизм, алгоритм.

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