SYNERGETICS AND THEORY OF CHAOS

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# CATEGORICAL COMPUTING SYSTEMS

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This paper is the second (first see [1]) in a series whose goal is to develop a fundamentally new way of constructing theories of cacomputing (categorical computing). The motivation comes from a desire to address certain deep issues that arise when contemplating quantum & mathematical biology's' theories of space and time. A topos is special type of the category. The topos approach to the formulation of informatics' theories includes a new form of informatics' logic. We present this topos informatics' logic, including some new results, and compare it to standard intuitionistic logic, all with an eye to conceptual issues. Importantly, topos informatics' logic comes with a clear geometrical underpinning.

*Key words:* computing, category, topos.

But the wrongs love bears will make
Love at length leave undertaking
'No, the more fools in do shake,
In a ground of so firm making,
Deper still they drive the stake.' [1]

*Sir Philip Sidney*

## 1. Introduction. CaComputers

The biological, neural, quantum, quantum, quantum-fields, social or cultural systems are complex natural computing informatics' systems. To study such systems, in the last fifteen years the authors have gradually developed the notion of CACOMPUTERS (CAM).

**CAM** have some common characteristics: they are **open** (exchange with their environment). **CAM** are self-organized. **CAM** are built up from a more large **hierarchy** of interacting complexity levels. **CAM** may **memorize** their experiences to adapt to various conditions through a change of response.

**Mathematics** is defined as the science (In contradistinction, Newton and Leibniz think a science is defined as Mathematics) at which rephrase any information in mathematical terms. Further, we have studied of the mathematical theory of a **categorical computing systems** (or **cacomputing systems**, **CAMS**).

A cacomputing system is a "set of iterative interacting neuronsimple categorical constructions", [1].

But the elements of a natural complex **CAMS** varies with time, for example learning new skills. Thus they cannot be studied using observables defined on a fixed space of a stationary phases and laws. So, we will give:

**A**$_1$) a successive configurations of **CAMS**, formed by its components and their interactions at a given time $t$ (mathematical model is a **time-category t-**$\kappa$),

**A**$_2$) a process of varies with time between these configurations (mathematical model is a **functor** $\Phi_{t\text{-}s}$ for pair time-categories **t-**$\kappa$, **s-**$\kappa$).

The components of **CAMS** are organized in a hierarchical structure **¥**, with several levels of

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[1] Ні, не кохання зміст буття
Кохання, лиш, буття продовжить.
Не може буть від смерті вороття,
Земля чекає всіх, нікого не тривожить,
Обов'язковий пункт кругу життя.

*Вільний переклад Valery Gritsak von Gröner*

**Pict. 1. A. Fomenko. Hierarchic**

complexity. Each level depends of distinct laws, but the interfaces between levels play an essential part.

A complex component is itself obtained by binding together a **sample**, which determines its internal organization. This a sample consists in a family of more elementary components with distinguished links between them.

The transition between successive configurations comes from the archetypal operations: "**birth**, **build, bind**, **death**, **partition**". It is modeled by the process of **systems complexification**. A sequence of systems complexifications can lead to the formation of components with strictly increasing orders of complexity, see fig. 1. This process describes the change resulting from the following operations:

**A. addition** of new elements,

**D. destruction** of components or their rejection in the environment,

**C. complexity** of samples into more complex components,

**PD. partition** and **decomposition** of higher order components.

In this paper we assume given abstract category $K$ (a **ensembles Ob(**$K$**)**) together with **relations Mor(**$K$).

Moreover, $(\mathbf{o_1} \to \mathbf{o_2})\in\mathbf{Mor}(K)$ mean that the ensemble $\mathbf{o_1}$ is a **subensemble** $\mathbf{o_2}$; while $(\mathbf{o_1} \perp \mathbf{o_2})\in\mathbf{Mor}(K)$ mean is a test which $\mathbf{o_1}$ passes but $\mathbf{o_2}$ and all its subensembles fail.

## 2. Glossary. Category & Logic

Let $\Gamma^1 = (\mathbf{V}(\Gamma^1),\mathbf{A}(\Gamma^1))$ and $\Gamma^2 = (\mathbf{V}(\Gamma^2),\mathbf{A}(\Gamma^2))$ are the orgraphs, where $\mathbf{V}(\Gamma^1)$, $\mathbf{V}(\Gamma^2)$ are vertex and $\mathbf{A}(\Gamma^1))$ and $\mathbf{A}(\Gamma^2)$ are arrows. A **homomorphism** $\mathbf{F}(\mathbf{F_1},\mathbf{F_2})$ from $\Gamma^1$ to $\Gamma^2$ are the two maps

$$\mathbf{F_1}: \mathbf{V}(\Gamma^1) \longrightarrow \mathbf{V}(\Gamma^2)\ ,\ \mathbf{F_2}: \mathbf{A}(\Gamma^1) \longrightarrow \mathbf{A}(\Gamma^2); \qquad (1)$$

here the maps $\mathbf{F_1}$, $\mathbf{F_2}$ such that

(i) $(\mathbf{F_1}(v_1),\mathbf{F_1}(v_2))\in\mathbf{A}(\Gamma^2)$;

(ii) $\mathbf{F_2}(\mathbf{e})\in\mathbf{A}(\Gamma^2)$

for $\forall(v_1,v_2)\in\mathbf{A}(\Gamma^1)$ and for $\forall e\in\mathbf{A}(\Gamma^1)$.

Usually, we refer us a single symbol $\mathbf{F} = \mathbf{F}(\mathbf{F_1},\mathbf{F_2})$. We will denoted by $Hom(\Gamma,\mathbf{G})$ the set of all **homomorphisms** from an orgraph $\Gamma$ to an orgraph **G**.

A **category** $K$ consists of two collections, **Ob(**$K$**)**, whose elements are the **objects** of $K$, and **Mor(**$K$) the **morphisms** (or **arrows**) of $K$. To each arrow is assigned a pair of objects, called a **source** and a **target** of the arrow.

The notation $\varphi: \mathbf{o_1} \longrightarrow \mathbf{o_2}$ means that as a morphism with source of the object $\mathbf{o_1}$ and target of the object $\mathbf{o_2}$.

If $\varphi_1: \mathbf{o_1} \longrightarrow \mathbf{o_2}$ and $\varphi_2: \mathbf{o_2} \longrightarrow \mathbf{o_3}$ are two morphisms, there is a morphism $\varphi_2 \circ \varphi_1: \mathbf{o_1} \longrightarrow \mathbf{o_3}$ called a **composite** of $\varphi_1$ and $\varphi_2$. For each object **o** there is a morphism $\mathbf{id_o}$, called an **identity** of **o**, whose source and target are both **o**. These data are subject to the following axioms:

A1) for $\varphi: o \longrightarrow u,\ \varphi \circ id_o = id_u \circ \varphi = \varphi$ ;

A2) for $\varphi_1 : \mathbf{o_1} \longrightarrow \mathbf{o_2}$, $\varphi_2: \mathbf{o_2} \longrightarrow \mathbf{o_3}$, $\varphi_3: \mathbf{o_3} \longrightarrow \mathbf{o_4}$, $\varphi_3 \circ (\varphi_2 \circ \varphi_1) = (\varphi_3 \circ \varphi_2) \circ \varphi_1$

Further, I offer some definition.

An **epimorphism** $\varphi$: $\mathbf{o} \to \mathbf{u}$ between objects **o** and **u**, in category, is a morphism such that for any pair $\varphi_1, \varphi_2 : \mathbf{u} \rightrightarrows \mathbf{o}$ of morphisms, the equality $\varphi_1 \circ \varphi = \varphi_2 \circ \varphi$ implies that $\varphi_1 = \varphi_2$. An epimorphism $\varphi$ is denoted by $\varphi : \mathbf{o} \twoheadrightarrow \mathbf{u}$.

A **monomorphism** $\varphi$: $\mathbf{o} \to \mathbf{u}$ between objects **o** and **u**, in category, is a morphism such that for any pair $\varphi_1, \varphi_2 : \mathbf{u} \rightrightarrows \mathbf{o}$ of morphisms, the equality $\varphi \circ \varphi_1 = \varphi \circ \varphi_2$ implies that $\varphi_1 = \varphi_2$. A monomorphism $\varphi$ is denoted by $\varphi : \mathbf{o} \rightarrowtail \mathbf{u}$.

A **terminal object** in a category is an object ∎ if for every object **o** in the category, there is one and only one morphism $\mathbf{o} \to$ ∎.

An **initial object** in a category is an object ⊙ if for every object **o** in the category, there is one and only one morphism ⊙ $\to \mathbf{o}$.

An **isomorphism** $\varphi$: $\mathbf{o} \leftrightsquigarrow \mathbf{u}$ is a a morphism $\varphi$ between the objects **o** and **u**, the category, if there is a morphism $\psi$: $\mathbf{u} \to \mathbf{o}$ such that $\varphi \circ \psi = \mathbf{id_u}$ and $\psi \circ \varphi = \mathbf{id_o}$.

**Isomorphic objects** denoted $\mathbf{o} \cong \mathbf{u}$ are said the objects **o** and **u**, if there is a morphism $\varphi$: $\mathbf{o} \to \mathbf{u}$ that is an isomorphism $\varphi$: $\mathbf{o} \leftrightsquigarrow \mathbf{u}$.

A **global element** of an object **o**, in a category, is defined to be a morphism $\varphi$: ∎ $\to \mathbf{o}$. The set of all global elements of **o** is denoted $\Gamma_{\mathbf{o}}$.

A diagram

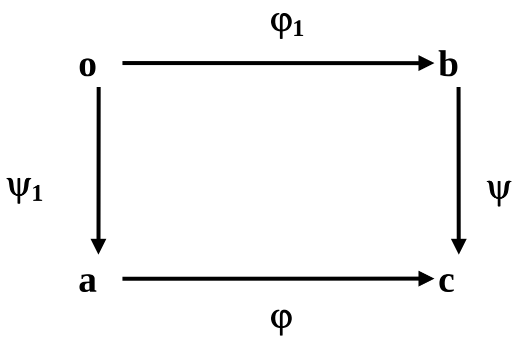
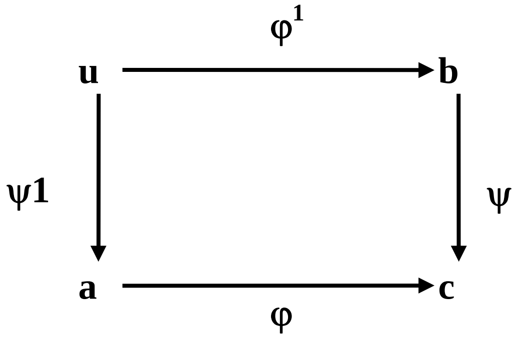
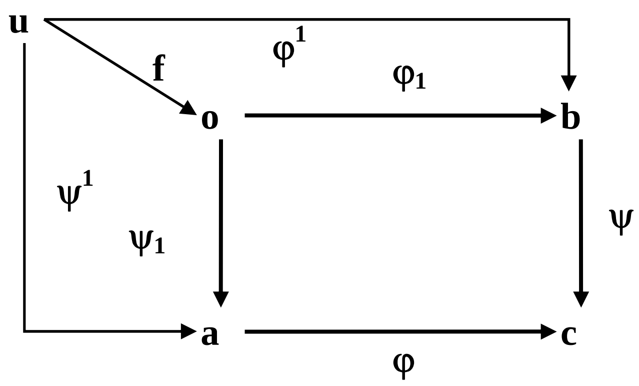



**Fig.1**

is called a **pullback square** provided that it commutes and that any commuting square of the form




There exists a unique morphism $\mathbf{f} : \mathbf{u} \to \mathbf{o}$ for which the following diagrams commutes

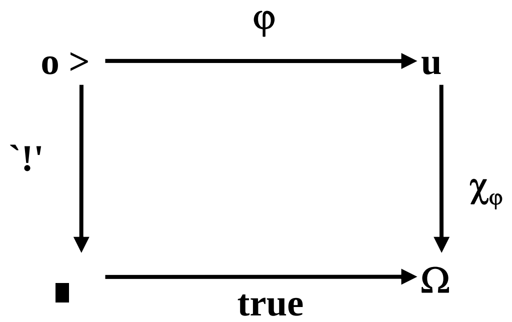



$\varphi_1$ is called a pullback of $\varphi$ along $\psi$.

Let ℍ be a set of monorphisms. A ℍ **-subobject** of an object **o** is a pair (**u** , $\varphi$), where $\varphi$: **u** → **o** belongs to ℍ.

Let K be a category with terminal object ∎. A **subobject classifier** for K is object **Ω** ∈ **Ob(**K**)** together with morphism, named **true**, **true** : ∎ ⟶Ω that satisfies the following axiom:

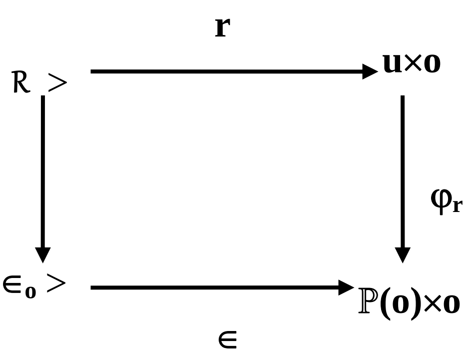
For each monomorphism $\varphi$: **o** >⟶ **u** there is one and only one morphism $\chi_\varphi$ such that




is a pullback square.

The morphism `!' denotes a unique morphism in the category. The morphism $\chi_\varphi$ is called the **characteristic morphism**, or **character of the monomorphism** (i.e., sub-object of **u**). The sub-object classifier, when it exists in b, is unique, up to isomorphism (i.e., there is an isomorphism between **Ω** and an object **u**).

Let K be a category with products "×". A category K is have **power objects** if to each object **o** there are an object ℙ**(o)** and an object $\in_{\mathbf{o}}$, and monomorphism ∈ **:** $\in_{\mathbf{o}}$ >⟶ ℙ**(o)** × **o**, such that for any object **u**, and monomorphism **r :** R → **u** × **o** there is exactly one morphism $\varphi_r$ : **u** → ℙ**(o)** for which there is a pullback of the form




Note in mend that in category theory it is morphisms, rather than the objects.

A **covariant functor** **Φ** : $K_1$ ⇒ $K_2$ from the category $K_1$ to the category $K_2$ is a function that assigns to each object **o**∈**Ob(**$K_1$**)** a object **Φ(o)**∈**Ob(**$K_2$**)**, and to each morphism $\varphi$: **o** ⟶ **u**, $\varphi$∈**Mor(**$K_1$**)** a morphism $\Phi(\varphi)$: $\mathbf{\Phi(o)} \longrightarrow \mathbf{\Phi(u)}$ in such that

*A.* $\Phi(\varphi_1 \circ \varphi_2) = \Phi(\varphi_1) \circ \Phi(\varphi_2)$, for the morphisms $\varphi_2$: $\mathbf{w} \longrightarrow \mathbf{o}$, $\varphi_1$: $\mathbf{o} \longrightarrow \mathbf{u}$, whenever $\varphi_1 \circ \varphi_2$ is defined

*B.* $\Phi_{id_o} = id_{\Phi(o)}$ for each object $o \in \mathbf{Ob}(\kappa_1)$.

A **contravariant functor** $\Psi : \kappa_1 \Rightarrow \kappa_2$ from the category $\kappa_1$ to the category $\kappa_2$ is a function that assigns to each object $\mathbf{o} \in \mathbf{Ob}(\kappa_1)$ a object $\mathbf{\Psi(o)} \in \mathbf{Ob}(\kappa_2)$, and to each morphism $\varphi$: $\mathbf{o} \longrightarrow \mathbf{u}$, $\varphi \in \mathbf{Mor}(\kappa_1)$ a morphism $\Psi(\varphi)$: $\mathbf{\Psi(u)} \longrightarrow \mathbf{\Psi(o)}$ in such that

*C.* $\Psi(\varphi_1 \circ \varphi_2) = \Psi(\varphi_1) \circ \Psi(\varphi_2)$, for the morphisms $\varphi_2$: $\mathbf{w} \longrightarrow \mathbf{o}$, $\varphi_1$: $\mathbf{o} \longrightarrow \mathbf{u}$, whenever $\varphi_1 \circ \varphi_2$ is defined

*D.* $\Psi_{id_o} = id_{\Psi(o)}$ for each object $o \in \mathbf{Ob}(\kappa_1)$.

For each category $\kappa$ the following conditions are equivalent:
(1) $\kappa$ is finitely complete,
(2) $\kappa$ has pullbacks and a terminal object.

A **presheaf** $\Re$ on a category $\kappa$ is a covariant functor $\Psi : \kappa \Rightarrow Set$, where $Set$ is category sets and sets functions.

A category $T$ is a **topos** if $T$ is finitely complete and has power objects.

With the term **non-classical logic** we refer to family of **logics** that differ from classical one, in various aspects. For example, a **intuitionistic logic** (**IL**) is a part of **classical logic** (**CL**), in the sense that all formulas provable in intuitionistic logic are also provable in classical logic.

Intuitionistic is here taken to mean that:

(IL) $\alpha \Rightarrow \neg\neg\alpha$

(CL) $\alpha \Leftrightarrow \neg\neg\alpha$

For those who wish to read more widely in category, graphs and logics see [2], [3].

### 3. Models of CAMS

Suppose $\kappa$ is a category such that
(i) = $\mathbf{Ob}(\kappa)$ are an objects;
(ii) $M = \mathbf{Mor}(\kappa)$ are a morphisms.

$O$ will be interpreted as CAMS elements (in quantum-field theory (q-f) - observation). The morphisms M will be interpreted as the interaction between the elements of CAMS.

A **maximal object** ⊞ of category

$$\kappa = (\mathbf{Ob}(\kappa), \mathbf{Mor}(\kappa)) \qquad (2)$$

is an unique terminal object $⊞ \in \mathbf{Ob}(\kappa)$ such that $\mathbf{o} \longrightarrow ⊞$ for $\forall \mathbf{o} \in \mathbf{Ob}(\kappa)/\{⊞\}$. The morphism

$$\mathbf{f}^{⊞} : \mathbf{o} \longrightarrow ⊞, \qquad (3)$$

is named **final morphism** of a element $\mathbf{o} \in \mathbf{Ob}(\kappa)$ (q-f interpreted : ⊞ - universe). and **initial object** ⊙ of $\kappa$ be named minimal object (q-f interpreted : ⊙ – black hole). The morphism

$$h^{\blacksquare} : \mathbf{o} \longrightarrow \blacksquare, \qquad (4)$$

is named **final morphism** of a element $\mathbf{o} \in \mathbf{Ob}(\kappa)$.

A **minimal object** ⊙ of category $\kappa$ = (**Ob**($\kappa$), **Mor**($\kappa$)) is an unique initial object ⊙ ∈ **Ob**($\kappa$) such that ⊙ ⟶ **v** for ∀ **v**∈**Ob**($\kappa$) /{ ⊙ }. The morphism

$$\psi_{\odot} : \odot \longrightarrow v, \qquad (5)$$

is named **zero morphism** of an element **v**∈**Ob**($\kappa$) (q-f interpreted : ⊙ - super black hole).

*3.1. CAMS - Model of a Orthospace.*

Let $\kappa$ be the category with the maximal object ⊞. We shall say that a triple

$$\mathrm{Or} = (\kappa, \perp, \boxplus) \qquad (6)$$

is called a orthospace if **Mor**($\kappa$) has a ortomorphisms denoted $o_1 \perp_{(o_1,o_2)} o_2$ (shorthand - $o_1 \perp o_2$) if for ∀o1,o2 ∈ **Ob**($\kappa$)

$$\perp_{(o_1,o_2)} : o_1 \longrightarrow o_2,$$

and the maximal element ⊞∈**Ob**($\kappa$) (will be denoted a **roof**) if the following conditions hold:

(i) on ∀ $o_1,o_2$∈**Ob**($\kappa$)/{⊞} if $\perp_{(o_1,o_2)} : o_1 \to o_2$, then $\perp_{(o_2,o_1)} : o_2 \to o_1$ in this case $\perp_{(o_1,o_2)} \circ \perp_{(o_2,o_1)} = id_{o_1}$;

(ii) for ∀**o**∈ **Ob**($\kappa$)/{⊞} ∄ the orthomorphism $\perp_{(o,o)}$ **: o ⟶o**;

(i) for ∀**o**∈ **Ob**($\kappa$) obtains ⊞⊥**o**.

*3.2. CAMS -Model of a Preordered Orthospace.*

Let Or is an orthospace with minimal elements ⊙∈**Ob**(Or) /{⊞}. A **Preordered Orthospace** is a triple

$$\mathrm{Pr} = (\mathrm{Or}, \perp, \odot) \qquad (7)$$

will be denoted ⊙ a **hol**) if the following condition hold:

(i) on ∀$\mathbf{o_1,o_2,o_3}$∈**Ob**(Pr) satisfying if $(\mathbf{o_1} \longrightarrow \mathbf{o_2}) \wedge (\mathbf{o_2} \perp \mathbf{o_3}) \Rightarrow (\mathbf{o_1} \perp \mathbf{o_3})$.

Consider further objects and morphisms a ortocategory $\kappa$.

First, sibling term in Q to a orthogonally ⊥ is the notion of a nearness (or a proximity), which indicated ≖.

$$o_1 \fallingdotseq o_2 \Leftrightarrow o_1 \not\perp o_2$$

for ∀$\mathbf{o_1,o_2}$∈**Ob**($\kappa$)/{⊙}. The concept of the **nearness** is a generalization the **identity**. Obviously, the nearness relation is **reflexively**, **symmetric** in a class objects **Ob**($\kappa$)/{⊙}. A nearness partition **Ob**($\kappa$)/{⊙} on equivalence classes, which will mark $E_v$ index every class one of his representatives **v**∈**Ob**($\kappa$)/{⊙}.

$\mathbf{o_1,o_2}$∈**Ob**($\kappa$)/{⊙}. $o_1$ **interacts** $o_2$ if

$$\exists f : o_1 \longrightarrow o_2, f \in \mathrm{Mor}(\kappa) / \{\perp\}. \qquad (8)$$

Denote $\mathbf{o_1} \geqslant_{\kappa} \mathbf{o_2}$ any interact relations (7) of the pair $(\mathbf{o_1}, \mathbf{o_2})$, or simply $\mathbf{o_1} \geqslant \mathbf{o_2}$.

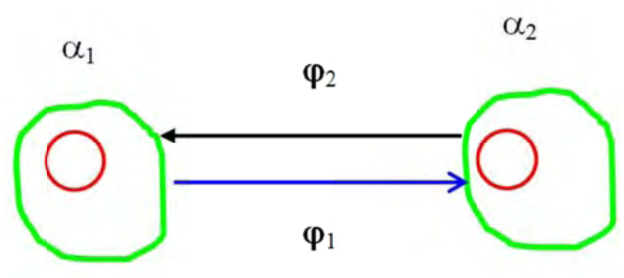



**Fig. 2.**

**Example.** A natural neurons system (**NNS**) consists of a single neurons and **NNS's** decision is composed of its neuron's decisions. Let a pair is a digraph $G = (V, A)$, where $V$ is a vertex (neurons) and $A$ is an arrows (neurons contact). In the digraphs modelling natural neurons systems, see the books [3–4], an energetic constraints are by the association to their arrows of a **weight** (real number) measuring their strength. Then the weight of a composite is a given function of the weights of its arrows. In this case, the category

$$K_n = (\mathbf{Ob}(K_n), \mathbf{Mor}(K_n)) \tag{9}$$

can be constructed as the quotient of the category of paths $\{L_i\}^N$ of an appropriate digraph by the relation which identifies two paths $L_i$ , $L_j$ having the same weight **N**. The objects of **Ob**($K_n$) are neurons. The morphisms of **Mor**($K_n$) are composite for all paths having the constant weight $\mathbf{N_0}$. A **signal** transmission be composite for all the morphisms. The categorical model of two neurons is by in figure 2. A signal transmission $\varphi_1$ is from a presynaptic neuron $\alpha_1$ to a postsynaptic neuron $\alpha_2$. If $\varphi_1$ is **no inhibition**, then all morphisms $\mathbf{f} \in \varphi_1$ are no a orthomorfism. If $\varphi_1$ is a **inhibition**, then morphism $\varphi_1$ is a orthomorfism to $\varphi_2$ and denoted by

$$\varphi_1 \perp \varphi_2. \tag{10}$$

$\odot \in$ **Ob**($K_n$) is a **input port** (receptors of neurosystem **NNS**).

**Theorem 1.** The triple

$$N_r = (K_n, \perp, \odot) \tag{11}$$

is a preordered orthospaces, when $K_n$ is the category (9), "⊥" are the orthomorphisms (10) and $\Theta \in \mathrm{Ob}(K_n)$ is input port of $K_n$.

**Pict. 3. A.Fomenko. The Neurocomputer**

**Proof**. In [2].

## 4. Dynamic Models of CAMS

### *4.1. CAMST - Model of a Timescale Preordered Orthospace*

Let $\mathcal{P}r$ is the preordered orthospace (7). Thus they cannot be studied using observables defined on a fixed space of a stationary phases and laws. So, **CAMST** is not represented by a unique category, will give:

**C**$_1$) a **timescale** $\mathcal{T}$ is a finite part of $\mathbb{R}$ and **t**$\in\mathcal{T}$ is called a **date**, where $\mathcal{T} \subset \mathbb{R}$, $|\mathcal{T}| < \infty$,

**C**$_2$) a **time-category t-**$\mathcal{K}$ for each of the dates **t**,

**C**$_3$) a relationship from the date $t_1$ to the date $t_2$ is a covariant relationship functor $\Phi_{t\text{-}s}$ for pair time-categories t-$\mathcal{K}$ , s-$\mathcal{K}$, where

$$\Phi_{t\text{-}s} : t\text{-}\mathcal{K} \Rightarrow s\text{-}\mathcal{K}, \qquad (12)$$

for each pair date (**s** , **t**), **s** > **t**. Furthermore, the relationship functors satisfying the condition**:**

**C**$_{3\text{-}1}$) if **v**,**s**,**t**$\in\mathcal{T}$ and if **v** > **s** > **t**, then the relationship functors $\boldsymbol{\Phi_{t\text{-}s} \circ \Phi_{s\text{-}v}}$, $\boldsymbol{\Phi_{t\text{-}v}}$ are equal.

## 5. Control System of CAMST

### *5.1. A pattern recognizer (PR)*

あ (♠) in a the time-category

$$\mathbf{t\text{-}\mathcal{K} = (Ob(t\text{-}\mathcal{K}) , Mor(t\text{-}\mathcal{K}) , \mathcal{T})} \qquad (13)$$

is the data of a orgraph

$$\mathbf{\Gamma = (V(\Gamma),A(\Gamma))} \qquad (14)$$

and a homomorphism **F(F**$_1$**,F**$_2$**)** from Γ to **t-**$\mathcal{K}$, where

$$\mathbf{F1: V(\Gamma) \longrightarrow Ob(t\text{-}\mathcal{K}),\ F2 : A(\Gamma) \longrightarrow Mor(t\text{-}\mathcal{K})} \qquad (15)$$

with

$$\mathbf{(F1(v1),F1(v2)) \in Mor(t\text{-}\mathcal{K});} \qquad (16)$$

$$\mathbf{F2(e) \in Mor(t\text{-}\mathcal{K})} \qquad (17)$$

for $\forall$**(**$v_1$**,**$v_2$**)**$\in$**A(Γ)** and for $\forall$**e**$\in$**A(Γ)**, and $\mathcal{T}$ is the timescale with the functor's properties **C**$_1$**- C**$_{3\text{-}1}$. The map **F**$_1$ (15) compare with a vertex **v** of Γ on an object $\mathbf{o_v}$ of **t-**$\mathcal{K}$, called an **object** $\mathbf{o_v}$ **of the pattern recognizer** (or **PR** ) あ, and an arrow **x** = <**v** , **w**> from **v** to **w** in Γ on a morphism $\varphi_x\in$**Mor(t-**$\mathcal{K}$**)** from $\mathbf{o_v}$ to $\mathbf{o_w}$ , called a **link** $\mathcal{L}(\varphi_x)$ **of the pattern recognizer** あ. A **multi-link** $\mathcal{L}((\varphi^i)$, **o) of the pattern recognizer** あ **to an object o**$\in$**Ob(t-**$\mathcal{K}$**)** is a family of morphisms ($\varphi_i$) from each $\mathbf{o_i}$ to **o**, correlated by the links of the pattern recognizer あ, that is, for each **y** = <$\mathbf{o_i}$ , $\mathbf{o_j}$>, we have $\varphi_i = \varphi_y \circ \varphi_j$ . Such a multi-link defines a **cone** with object **o** and the pattern recognizer あ as its **basis**.

An object **o*** of the pattern recognizer あ(♠) is a **possible colimit** (**pc**), pc denoted $\mathcal{C}^o$(あ→**o***) if there exists an object **o***$\in$**Ob(t-**$\mathcal{K}$) satisfying the following conditions:

(**1cl**) $\exists$ a noted multi-link $\mathcal{L}$**(**($\psi^i$), **o*)** of the **PR** あ to an object **o***, and

(**2cl**) ∀ multi-link $LC^o((\gamma^i), \mathbf{o}^\#)$ of the **PR** あ to an object $\mathbf{o}^\#$ is bound into a unique morphism φ from **o*** to $\mathbf{o}^\#$.

A **PR** あ is called a **decomposition** $\nabla_{あ}(\mathbf{o}^*)$ (see **introduction**) of object **o*** if exists unique a **pc** $C^o$(あ→**o***).

*5.2. Categorical Scheme-Model of Organism. SCOM*

Further, a sextuple

$$DC = (Pr, \mathbf{t\text{-}K}, T, あ, \Gamma, \mathbf{F(F_1,F_2)}) \quad (18)$$

is called a **Dynamical CAMST** (or a **Categorical Scheme-Model** of **Organism** (**SCOM**)), where $Pr = (Or, \perp, \odot)$ is a preorderd orthospaces and $Or = (\mathbf{t\text{-}K}, \perp, \boxplus)$ is an orthospace, where $\mathbf{t\text{-}K} = (\mathbf{Ob(t\text{-}K)}, \mathbf{Mor(t\text{-}K)}, T)$ is a time-category; here $T$ is timescale, あ is a **PR** such that

(a) the time-category **t-K** is the data of a orgraph $\Gamma = (\mathbf{V}(\Gamma),\mathbf{A}(\Gamma))$;

(b) $\mathbf{F(F_1,F_2)}$ is a homomorphism from Γ to **t-K**, such that the conditions (15) - (17) hold.

A orgraph $\Gamma = (\mathbf{V}(\Gamma),\mathbf{A}(\Gamma))$ of **COM** is called a **graph-scheme** (or **graph-control**) of SCOM. SCOM is operable at several graph-schemes, see in figure 4.

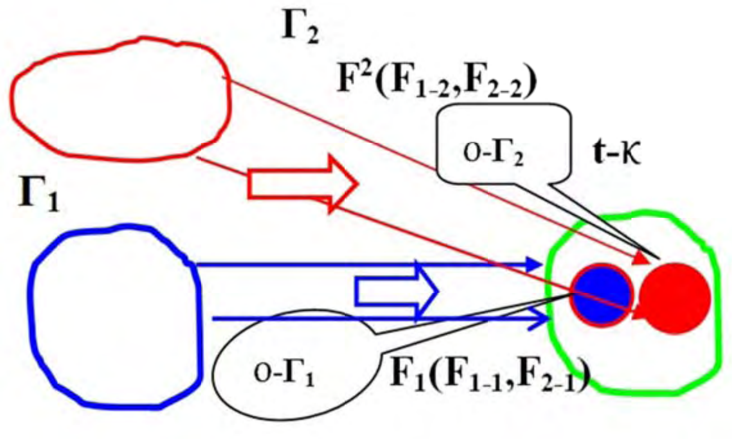



**Fig. 4.**

In fact, **SCOM** is the sextuple

$$DC = (Pr, t\text{-}K, T, あ_i, \{\Gamma_i\}, F^i(F_{1\text{-}i},F_{2\text{-}i})) \quad (18)^*$$

is called a **Dynamical CAMST** (or a **Categorical Scheme-Model** of **Organism**), where $\{\Gamma_i\}$, $\mathbf{i} = [\mathbf{1},\mathbf{n}]$ are graph-controls. A **action-field** $O\text{-}\Gamma_i$ of the graph-scheme $\Gamma_i = (\mathbf{V}(\Gamma_i),\mathbf{A}(\Gamma_i))$ is the minimal subcategory of **t-K** with $\mathbf{Ob}(O\text{-}\Gamma_i) \supseteq \mathbf{F_{1\text{-}i}}(\mathbf{V}(\Gamma_i))$ and $\mathbf{Mor}(O\text{-}\Gamma_i) \supseteq \mathbf{F_{2\text{-}i}}(\mathbf{A}(\Gamma_i))$.

*5.3. Books and Complexity of SCOM*

Suppose the simple links from PR $あ_1$ to PR $あ_2$ come from interactions between the objects of these two PR; then they are modeled by the notion of a **singleton**. A **singleton** $S(あ_1, あ_2)$ from PR $あ_1$ to PR $あ_2$ is a maximal set of morphisms (called **darts** of the singleton $S(あ_1, あ_2)$) between their objects: $\mathbf{Ob}(O\text{-}\Gamma_1)$ and $\mathbf{Ob}(O\text{-}\Gamma_2)$ if satisfying the following conditions:

(**s1**) For ∀ object **o**∈Ob(O-$\Gamma_1$), there exists at least one dart of the singleton S(あ$_1$,あ$_2$) from **o** to an object of **Ob**(O-$\Gamma_2$), and if there exist several such darts, they are edged by a loop of links L(**$\varphi_o$**) of the **PR** あ$_2$, here $\varphi_o$∈**Mor**(O-$\Gamma_2$);

(**s2**) The morphisms obtained by composing a dart of the singleton with the link of PR あ$_1$ on the left, or of PR あ$_2$ on the right are in the singleton S(あ$_1$,あ$_2$).

Further, a singleton from PR あ$_1$ to PR あ$_2$ binds into a **unique morphism** $\varphi$(**o$_1$o$_2$**)∈**Mor**(**t**-K) from **o$_1$**∈**Ob**(O-$\Gamma_1$) to **o$_2$**∈**Ob**(O-$\Gamma_2$), called a (あ$_1$,あ$_2$)-**simple** respectively in the categories O-$\Gamma_1$ and O-$\Gamma_2$.

**Theorem 2.** Let **o$_1$**∈**Ob**(O-$\Gamma_1$), **o$_2$**∈**Ob**(O-$\Gamma_2$), **o$_3$**∈**Ob**(O-$\Gamma_3$). Suppose the morphism $\varphi$(**o$_1$o$_2$**) is (あ$_1$,あ$_2$)-simple and if the morphism $\varphi$(**o$_2$o$_3$**) is (あ$_2$,あ$_3$)-simple; then their composite $\varphi$(**o$_1$o$_2$**) ∘ $\varphi$(**o$_2$o$_3$**) is (あ$_1$,あ$_3$)-simple.

**Proof** is in [2],[5]-[7].

Let ␦ and ␧ are two pattern recognizers in SCOM (18)*. Then decomposition ∇␦(**o**) and ∇␧(**o**) of an object **o**∈**Ob**(O-$\Gamma_k$), $\Gamma_k$ ∈{$\Gamma_i$}, are said to be **equivalent** if there exists a singleton S(␦, ␧) from ␦ to ␧ connection of **o** such that this connection is a (␦,␧)-simple dart. An object **o** is **book** (or **cat-manifold**) if it present at least two non-equivalent decompositions ∇␦(**o**) and ∇␧(**o**).

Now, (see introduction) we shall say that a **complexity m** of SCOM (18)* is called the length of a maximal pairwise different chain of pattern recognizers

$$\Xi_1, \ldots, \Xi_i, \Xi_{i+1}, \ldots, \Xi_m \qquad (19)$$

such that $\nabla_{\Xi_i}$ and $\nabla_{\Xi_{i+1}}$ are non-equivalent decompositions.

**Example.** In natural biological systems (for example, biological organisms) consists of an organs, the pattern recognizer play an important part (see the paper [4]).

### 6. Hierarchical CaComSystems

We will use terms from previous paragraph.

If $C^o$(あ→**o**) is the **pc** of a **PR** あ of linked L(($\varphi^j$),**o$_i$**) objects **o$_i$**∈**Ob**(**t**-K) and if each **o$_i$** is the **pc** $C^o$(あ$_i$→ **o$_i$**) of a **PR** あ$_i$ , we say that **o** is a **second-branchification** of (あ,(あ$_i$)), or that (あ,(あ$_i$)) is a **branchification** of **o** of **length 2**.

Further, we define:

A **n-iterated possible colimit** (or **n-pc**) **o** is the **pc** $C^n$(あ→**o**) of a pattern recognizer あ each object of which is itself a (**n -1**)-iterated colimit. A **n-branchification** of **o** is the data of a decomposition ∇あ(**o**) of **o** and of a (**n-1**)**-branchification** of each component of this decomposition.

Now a category **t**-K is **hierarchical** if its objects **Ob**(**t**-K) are partitioned in a sequence of complexity levels **0,1,...,m** (see (19)), so that an object **o**∈ **Ob**(O-$\Gamma_k$), $\Gamma_k$ ∈{$\Gamma_i$},of level **m+1** is the **pc** of at least one the **PR** formed by linked objects **o$_i$** of level **m**.

A morphism **f**∈**Mor**(**t**-K) between two objects **o$_1$**,**o$_2$**∈**Ob**(**t**-K) of level **m +1** is called an **n-simple link** if **f** is a dart of a singleton S(あ$_1$,あ$_2$)) between two **PR** あ$_1$ and あ$_2$ of level less or equal to **m.** A morphism **f**∈**Mor**(**t**-K) is a **n-complex link** if it is the composite of **m**-simple darts non-adjacent singletons, so that it is not n-simple.

A **m-book** (or **m-catmanifold**) is called if for each **m**, there exist objects **o**∈ **Ob**(O-$\Gamma_k$), $\Gamma_k$ ∈{$\Gamma_i$}, of level **m+1** which admit non-equivalent branchification of level **m**.

**Theorem 3.** The existence of **m**-complex links necessitates that the category **t**-K satisfies:

(i) For each **m**, there exist **m**-books **b**∈**Ob**(O-$\Gamma_k$), $\Gamma_k$ ∈{$\Gamma_i$}, of level **m+1**.

(ii) An object **o**∈**Ob**(O-$\Gamma_k$), $\Gamma_k$ ∈{$\Gamma_i$}, of level **m** can participate in several **PR** あ having different **n-pc** at level **m+1**.

**Proof** is in [2],[5]-[7].

A **height of complexity** of an object **o**∈**Ob**(O-$\Gamma_k$), $\Gamma_k$ ∈{$\Gamma_i$}, is the smallest **h** such that there exists a pattern of linked objects of level **h** with **o** as its possible **h** -colimit $C^h$ (あ→**o**). And **o** is **r-reducible** for each **r** greater than or equal to its order.

**Theorem 4.** An object **o**∈**Ob**(O-$\Gamma_k$), $\Gamma_k$ ∈{$\Gamma_i$} of level **m+1** which admits a decomposition ∇ฎ(**o**) of level **m** (see (19)) in which all the distinguished links multi-link L**(**($\varphi^i$), **o)** are (**m** -1)-simple is (**m** -1)-reducible.

**Proof** is in [5].

But on the other hand exist next the theorem.

**Theorem 5.** An object **o**∈**Ob**(O-$\Gamma_k$), $\Gamma_k$ ∈{$\Gamma_i$} will not be (**m** -1)- reducible if its decompositions ∇ฎ(**o**) of level **m** have some of their multilink's L**(**($\varphi^i$), **o)** which are not (**m** -1)-simple.

**Proof** is in [5].

Moreover, the next theorem is important.

**Theorem 6.** A category **t**-K from DC **(18)*** is a topoi **T**.

**Proof** is in [7].

### 7. Compound Hierarchical CaComSystems

In a CaComSystems, the transitions come from a complexification process with respect to a strategy.

A strategy St on a SCOM DC = (Pr, t-K, T, あ$_i$, {$\Gamma_i$}, $F^i(F_{1\text{-}i},F_{2\text{-}i})$)
is the quintuple

$$St = (\mathbf{V,O,D,L,P}) \qquad (20)$$

consists:

- a set **V** of "external elements" vertices of the graphs {$\Gamma_i$};
- a set **O** of objects of the category **t**-K;
- a set **D** of the decompositions ∇ฎ(**o**), **o**∈**O** without a colimit of level **1**;
- a set **L** of a multi-link L**(**($\varphi^i$), **o)**;
- a set **P** of **PR** あ$_i$ with a **pc** to the decompositions ∇ฎ(**o**).

The **Compound Hierarchical** (**CH**) CH of **CaComSystems** DC (18)* with respect to the strategy St (20) is building CaComSystems DC* in which St are realized in the most minimal way. In other words, DC* no contains DC* is building CaComSystems DC* in which St are realized. The illustration of CH sees in figure 5.

An important result is the next theorem.

**Theorem 7.** The reduction process from theorems 3-4 with strategy St (20) cannot be reduced to a unique a compound hierarchical CH for DC (18)* with nontrivial components in (18)*.

**Proof** is in [5–7].

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## Категорные вычислительные системы

Эта статья является второй (первая — [1]) в ряду, цель которого состоит в том, чтобы развить существенно новый способ построения теории категорных вычислений. Обращение к этой теме вызвано желанием рассмотреть определенные глубокие проблемы, возникающие при изучении теорий пространства и времени в квантовой и математической биологии. Топос — особый тип категории. Топосный подход к формулировке теорий информатики создает новую форму логики в информатике. Мы представляем логику этой топосной информатики, включая некоторые новые результаты, и сравниваем ее со стандартной интуиционисткой логикой, обращаясь к концептуальным проблемам. Важно, что топосная логика информатики идет с ясным геометрическим подкреплением.

*Ключевые слова*: вычисление, категория, топос.

**Pict 5. G. Klimt. Hierarchic of Life**