SYNERGETICS AND THEORY OF CHAOS

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HAOTICS AND BIOCOMPUTING APPLICATION

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The idea of a "Combinatorial Chaotics" or Chaotic was as is well known, originally suggested by V. V. Gritsak-Groener in his pioneering article [1]. In this article we construct the model combinatorial chaotic of a collection of bioinformatics objects:

a) a flows in a chaos;
b) a chaos in a permutation;
c) a chaos in combinatorial configurations.

We also construct the computational algorithms of the problems a)-c).

Key words: chaos, synergetics, bioinformatics, algorithm of chaos.

1. Flows in Chaos

Consider a digraph

$$\Gamma = (V(\Gamma), E(\Gamma), v^+, v^-, \varphi),$$

where $E(\Gamma)$ is the arc-set, $V(\Gamma)$ is the vertex-set containing a source $v^+ \in V(\Gamma)$ and sink $v^- \in V(\Gamma)$, and $\varphi: E(\Gamma) \longrightarrow R^+$ is the function defining the *capacity of arcs*. Let

$$\mathcal{P} = \{P \subset V(\Gamma) : v^+ \in P, v^- \notin P\}.$$

For $P \in \mathcal{P}$, we refer to

$$R(P) = \{e \in E(\Gamma) : \partial^+ e \in P, \partial^- e \notin P\}$$

as the cut corresponding to P and define its capacity by

$$\varphi(P) = \sum_{i=1} \varphi(e_i), \; e_i \in R(P), \; r = |R(P)|.$$

Definition 1. A *flow* in Γ is a function

$$\Theta : E(\Gamma) \longrightarrow R^+$$

that satisfies capacity condition:

$$0 \le \Theta(e) \le \varphi(e)$$

for each $e \in E(\Gamma)$ and the conservation condition:

$$\Theta(\delta^+ v) = \Theta(\delta^- v)$$

at each vertex $v \in V(\Gamma)$ distinct from e^+ and e^-, where

$$\Theta(\delta^+ v) = \sum_{\forall e \in \delta^+ v} \Theta(e) \text{ and } \Theta(\delta^- v) = \sum_{\forall e \in \delta^- v} \Theta(e).$$

The *max- flow problem* is to maximize the value of flow Θ.

Theorem 1. The maximum value of a flow Θ in the digraph Γ is equal to the minimum capacity of a cut.

Corollary 1.1. Efficient algorithms of complexity such as $O(|V(\Gamma)|^3)$ are known for finding a maximum flow.

The proof and the algorithm are found in [1].

The graph-theoretical notion of network flows has been generalized to combinatorial configurations and in this note we make a natural extension of it to a pair of clutters on the ground set A. We shall give a short summary of some of main results known about network flows on combinatorial configurations and chaotics.

The basic definitions of the terms pertaining to flows in chaos are as follows.

Definition 2. Let $\Im = (A,C)$ be finite chaotic with the groundset $A = \{a_0,a_1, . . .,a_n\}$ and the cycles $C = \{c_1,c_2, . . .,c_d\} \subset 2^A$. $a_0 \in A$ and is called the *flows with input in* a_0 of $\Im$. Given $C(a_0) \subseteq C$, $C(a_0) = \{\alpha_i \in C : \forall \alpha_i \ni a_0\}$, i=[1,r]. And given $V = \{v_i \in Q^+ : i=[1,n]\}$. v_i is called *weight* of the element $a_i \in A$. Further, given the matrix $M = [\tau_{ij}]_{n\times r}$, where $\tau_{ij} = 1$ if $a_i \in \alpha_j$. Since $a_i \notin \alpha_j$, we have $\tau_{ij} = 0$. The matrix M is called *flow-matrix across the cycles* $C(a_0)$. Finally, define the flows $\mathcal{F}_\Im$ of $\Im$ by

$$(p_1,p_2, . . .,p_r),\ p_i \in Q^+,$$

Where

$$\sum_{j=1} \tau_{ij} p_j \le v_i,\ i=[1,n].$$

$\mathcal{V}_\Im = \sum_{j=1} p_j$ is called *value* of flows $\mathcal{F}_\Im$.

Examples 1. A blood flow, a limphe flow, a toxic flux, a peniciline propagation and other are biological&medicine examples for flow in chaos.

Definition 3. Let $\Im = (A,C)$ be chaotic in definition 1.
We assume that

$$(x_1^0, ..., x_r^0)$$

is the solution of the problem

$$P(\Im) = \sum_{j=1} x_j \longrightarrow \max$$

$$\sum_{j=1} \tau_{ij} x_j \le v_i,\ i=[1,n].$$

The vector $(x_1^0, ..., x_r^0)$ is a *maximum* $\Im$-flow in the presence of weights $V = \{v_i \in Q^+ : i=[1,n]\}$ and flows with input in $a_0 \in A$. If c_k is cycle of $\Im$ which contains a_0 then by the *capacity* $f(c_k)$ of c_k (with respect to a_0) we mean

$$f(c_k) = \sum_{i|a_i \in c_k} v_i .$$

We say that the chaotic $\Im = (A,C)$ is called a *regular* if for each $a_0 \in A$ which is not a loop of $\Im$ and for any set of capacities $V=\{v_1, ...,v_n\}$ the value of the maximum $\Im$-flow equals the minimum capacity $f^{min}(c_k)$ of a_0, i. e.

$$P^{max}(\Im) = f^{min}(c_k).$$

Examples 2. (See fig.1). The picture is the illustration of the chaotic-flow.
Next, we are now in a position to state the problem of chaotic theory.

Problem JULIA. Let $\Im = (A,C)$ is a finite chaotic. Where $\Im$ is a regular chaotic?

Theorem 2. Let $\Im = (A, C)$ is a finite chaotic. $a_0 \in A$ is not a loop and for any set of capacities $V=\{v_1, ...,v_n\}$ ($\forall v_i \ge 0$) the value of the maximum $\Im$-flow equals the minimum capacity $f^{min}(c_k)$ of a_0, i. e.

$$P^{max}(\Im) \le f^{min}(c_k).$$

The proof and the algorithm are found in [2]. Detailed biologic interpretation sees in [3].

2. Recognition Algorithm of Strength Chaotic Height

At a basic level, one would expect that biological chaos would address the question of how one constructs the chaotic height for fundamental objects in biology. As usual, biological fundamental objects (the amino acid sequences of proteins, the neuron sequences) are strength of bio-objects (*b*-objects). Any such list of objects gives the objects of a linear order. These mean that if α and β are *b*-objects, then we can say that $\alpha < \beta$ if α precedes β on the list. A structure different from linear order early gives a chaotic combinatorial configuration on b-objects. If the list has *n b*-objects, the linear order must map these objects to the set

$$1, 2, 3, \ldots, n.$$

Furthermore, one might expect that the linear order have chaotic height of strength (chs) null.

Fig. 1. W. Kandynsky, "Flow".

Definition 4. A *permutation* of *n* distinct *b*-objects of length *i* is an ordered arrangement of any *i* of the *b*-objects and is denoted Π_n^i. By P(*n*) denote the permutation of *n* distinct *b*-objects of length *n*. A permutation P(*n*) is frequently called a permutation of *n*.

Examples 3. For example, the permutation of $S_4 = \{\alpha,\beta,\chi,\delta\}$ of length 2 are $\alpha\beta$, $\alpha\chi$, $\alpha\delta$, $\beta\alpha$, $\beta\chi$, $\beta\delta$, $\chi\alpha$, $\chi\beta$, $\chi\delta$, $\delta\alpha$, $\delta\beta$, and $\delta\chi$.

Theorem 3. The number of permutations of *n* b-objects of length *i* is

$$\mu(P(n)) = n \times (n-1) \times \ldots \times (n-i+1).$$

The proof is given in exercises.

Definition 5. Let P(*n*) is a permutation n. A *strength chaotic height* (sch is denoted $\lambda(P(n))$ of P(*n*) is equal to $\lambda(P'(n-1))\times n + (i_0 - 1)$ if n is odd number and sch is equal to $\lambda(P'(n-1))\times n + (n - i_0)$ if *n* is even number, where $P'(n-1)$ is a permutation $n-1$ is equal to P(*n*) without *n* and i_0 is position *n* in P(*n*). $\lambda(P(1)) = 0$.

A *relative strength chaotic height* (rsch is denoted $\chi(P(n))$) of P(*n*) is equal to $\lambda(P(n) : n!$, where $n! = 1 \times 2 \times \ldots \times n$.

Examples 4. For example, we computation $\lambda((2,3,1,5,4))$ and $\chi((2,3,1,5,4))$. We have $\lambda((1)) = 0$. Therefore, we have

$$\lambda((1)) = 0 \Rightarrow \quad \lambda((2,1)) = 0\times 2 + (2-1) = 1;$$
$$\lambda((2,1)) = 1 \Rightarrow \quad \lambda((2,3,1)) = 1\times 3 + (3-2) = 4;$$
$$\lambda((2,3,1)) = 4 \Rightarrow \quad \lambda((2,3,1,4)) = 4\times 4 + (4-4) = 16;$$
$$\lambda((2,3,1,4)) = 16 \Rightarrow \quad \lambda((2,3,1,5,4)) = 16\times 5 + (4-1) = 83.$$

Finally, we obtain $\chi((2,3,1,5,4)) = 83/120 \approx 0.69167$.

Theorem 4. If P(*n*) is a permutation n, then

$$0 \le \lambda(P(n)) < n!.$$

The proof is trivial.

Corollary. If P(*n*) is a permutation n, then $0 \le \chi(P(n)) < 1$.

The **sch** and **rsch** algorithm is listed in Algorithm 1. If necessary operate with a large numbers, then see an algorithm in [5, 6].

```
/******************** Algorithm 1. **********************************
challenge:
#include "heiperm.cpp"
void main()
{
  int perm[6]={0,2,3,1,5,4};
  int lambda=hperm(5,perm);
```

```
  double hi=rhperm(5,perm);
};
*********************************************************/
int hperm(int n, int *perm)
// Commentary 1
{
  if (n==1) return 0;
  else
  {
    int* perm1; // Commentary 2
    int j;
    int i0; // Commentary 3
    perm1=new int[n]; // Commentary 4
    j=1;
    for (int i=1; i<=n; i++)
    {
      if (perm[i]!=n)
      {
        perm1[j]=perm[i]; j++;
      }
      else i0=i;
    };
    int lpn1=hperm(n-1,perm1); // Commentary 5
    delete [] perm1; // Commentary 6
    if ((n%2)==1) // Commentary 7
    return lpn1*n+(i0-1);
    else return lpn1*n+(n-i0);
  };
};

double rhperm(int n, int *perm)
// Commentary 8
{
  long f=1;
  for (long i=1; i<=n; i++) f=f*i; // Commentary 9
  double hp=hperm(n,perm);
  return hp/f;
};
```

3. Combinatorial Configurations. Prolog of Dynamics Methods

A man growth is the cell's (combinatorial configurations!) phase transitions:
gamete → embryo → snake → fish → animal → children → man(woman) → gamete.

Definition 6. Further, let

$$K = (A, B),\ B \subseteq 2^A$$

be a combinatorial configurations. Suppose $\Gamma_K = (V, E)$ is a K-*hypergraph* (or simple a hy-pergraph) with *hypervertex* set V = B, *hyperedge* set $E \subseteq B \times B$ and ground set A. We write hyperedges as $e = (b_1, b_2)$, when $b_1, b_2 \in B$ and $b_1 \cap b_2 \neq \varnothing$. If b_1 and b_2 are the end vertices, we call them *adjacent* and write $b_1 \leftrightarrow b_2$. The elements of the ground set A are called *microelements* Γ_K. All K-hypergraphs Γ_K (and the combinatorial configurations K) are assumed without further comment to be connected and finite, let K = (A, B) is a chaotic.

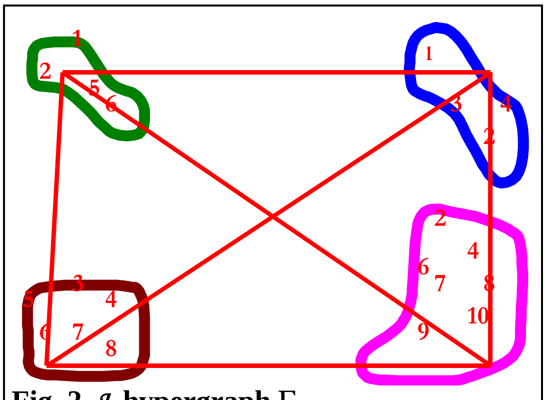

Fig. 2. $\mathcal{J}$**-hypergraph** $\Gamma_{\mathcal{J}}$**.**

Examples 5. (See Figure 2) The $\mathcal{J}$-hypergraph $\Gamma_{\mathcal{J}}$ is uniquely determined by Figure 2.

Let K = (A, B) is a chaotic. $\Gamma_K = (V, E)$ is a K-hypergraph. Given $T \subset V$, set $\aleph_V^T = \{V \ni v \notin T :$

∃t∈T, t ↔ v} is called the *crown* of T and $\mathcal{S}_V^T$={e=(v₁,v₂)∈E : v₁∈T, v₂∉T} is called the *igl* of T. Define the *crown-saturation* of Γ_K by

$$S\square_V^T = \min_{T\subset V} \{\mu(\square_V^T):\mu(T)\},$$

and let the *igl-saturation* of Γ_K by

$$S\square_V^T = \min_{T\subset V} \{\mu(\square_V^T):\mu(T)\}.$$

A hypergraph Γ_K is called ε-*igllable* if $S\mathcal{S}_V^T$=ε and a hypergraph Γ_K is called β-*crownable* if $S\mathcal{K}_V^T$ = β, where ε,β∈(0,1). An automorphism of Γ_K is a bijection f: V ⟶ V that induces a bijection of φ_f: E ⟶ E. The set of autorphisms of Γ_K forms a group denoted $\mathcal{A}ut(\Gamma_K)$. We say that a subgroup G ⊆ $\mathcal{A}ut(\Gamma_K)$ is *transitive* if for ∀v_1,v_2∈V, there is some g∈G such that $g(v_1) = v_2$. We call the hypergraph Γ_K *itself transitive* if $\mathcal{A}ut(\Gamma_K)$ is.

Theorem 5. Z_{i_k} be a cyclic group of , $n,m\in Z$. If G be a finitely generated commutative group and $\mathcal{J}$ a finite generating set for G, then

$$G \cong \overbrace{Z \oplus \ldots \oplus Z}^{n} \oplus \overbrace{Z_{i_1} \oplus Z_{i_2} \oplus \ldots \oplus Z_{i_m}}^{m},$$

$n + m = \mu(\mathcal{J})$.

The proof of theorem 5 and detailed information are in [7].

Definition 7. Let G be a finitely generated commutative group and $\mathcal{J}$ = {j_1, …, j_n,c_1, …, c_m} be a finite generating set for G, see (0.2.34). The *Groener hypergraph* (G-*hypergraph*) $\Gamma^G = \Gamma(G, \mathcal{J})$ of is a K-hypergraph with microelements set A = G and hypervertex set V = {v = gj : g∈G, j∈$\mathcal{J}$}, and hyperedge set E contains a pair (v_1,v_2), v_1,v_2∈V, $v_1 \cap v_2 \neq \varnothing$.

Examples 6. A cell bisection process is the phase transition of $\mathcal{J}$ = {c_1, c_2}.

A «gamete → embryo» process is the phase transition of $\mathcal{J}$ = { j_1, c_1, c_2}.

A «animal → children» process is the phase transition of $\mathcal{J}$ = { j_1, c_1, …, c_m}.

A «children → man(woman)» process is the phase transition of $\mathcal{J}$ = {c_1, …, c_m}.

Furthermore, a locally compact group G has a unique invariant σ-finite Radon measure $|*|^\Theta$; it is unique up to multiplicative constant [7]. For ∀g∈G, the measure S → $|g(S)|^\Theta$ is invariant, whence there is a positive number d^g such that $|g(S)|^\Omega = d^g \times |S|^\Theta$ for all measurable S. The map g ↦ d^g is inducers homomorphism from G to the multiplicative group of the positive reals and is called the *modular function* of G. If d^g = 1 for ∀g∈G, then G is called *unimodular*. We call a hypergraph Γ_K *unimodular* if Aut(Γ_K) is.

Theorem 6. We give the group $\mathcal{A}ut(\Gamma_K)$ of a hypergraph Γ_K the topology of Microscope method. Suppose there is a transitive unimodular closed subgroup G* of $\mathcal{A}ut(\Gamma_K)$, then $\mathcal{A}ut$(G*) is also unimodular.

The proof of theorem 6 and detailed of Microscope method are in [5], [7] and [8].

Therefore let K = (A,B) is a chaotic and Γ_K = (V,E) is a K-hypergraph. The stabilizer $\mathcal{S}t$(v) = {g∈$\mathcal{A}ut(\Gamma_K)$, v∈V : g(v) = v}, of ∀v∈ V is compact and so has Grit measure [7]. Note that if g(v*) = w, then $\mathcal{S}t(w) = (g(\mathcal{S}t(v^*)))\, g^{-1}$, whence $|\mathcal{S}t(w)|^\Theta = |(\mathcal{S}t(v^*))\, g^{-1}|^\Theta = (d^g)^{-1}\, |(\mathcal{S}t(v^*))|^\Theta$.

Theorem 7. A K-hypergraph Γ_K = (V,E), K = (A,B) is unimodular ⇔ for ∀v_1,v_2∈V = B in the same orbit, $|\mathcal{S}t(v_1)|^\Theta = |\mathcal{S}t(v_2)|^\Theta$.

Corollary 7.1. If Γ_K = (V,E), is transitive, then Γ_K is unimodular ⇔ $|\mathcal{S}t(v_1)|^\Theta = |\mathcal{S}t(v_2)|^\Theta$ for all neighbors v_1 and v_2, v_1,v_2∈V.

The proof is in [7].

Unimodularity of $\mathcal{A}ut(\Gamma_K)$, Γ_K = (V,E), K = (A,B) is finite combinatorial property.

Theorem 8. Let Γ_K = (V,E), K = (A,B) is a K-hypergraph. Let $|*|^\Theta$ denotes cardinality for subsets V and Hrit measure for subsets $\mathcal{A}ut(\Gamma_K)$. Then for any vertices v_1,v_2∈V,

$$|(\mathcal{S}t(v_1))v_2|^\Theta \,/\, |(\mathcal{S}t(v_2))v_1|^\Theta = |\mathcal{S}t(v_1)|^\Theta \,/\, |\mathcal{S}t(v_2)|^\Theta;$$

thus, Γ_K is unimodular $\Leftrightarrow$ for $\forall v_1, v_2 \in V = B$ in the same orbit,

$$|(\mathcal{S}t(v_1))v_2|^{\Theta} = |(\mathcal{S}t(v_2))v_1|^{\Theta}.$$

Theorem 9. Let $\Gamma_K = (V,E)$, $K = (A,B)$ is a K-hypergraph. If Γ_K is transitive, then Γ_K is unimodular $\Leftrightarrow$ (0.2.36) holds for all neighbors for any vertices $v_1, v_2 \in V$.

Finally, let G be the finitely generated commutative group and $\mathcal{L}(G)$ be the space of measurable real-valued functions on G that are essentially bounded with respect to the measure. A linear functional on $\mathcal{L}(G)$ is called a babaj if it maps the constant function to the number 1 and non-negative-functions to non-negative numbers. If $\varphi \in \mathcal{L}(G)$ and $g \in G$, we write $\mathcal{L}_g(\varphi(x)) = \varphi(g(x))$. We call a *babaj* $\mathfrak{X}$ invariant if $\mathfrak{X}(\mathcal{L}_g(\varphi)) = \mathfrak{X}(\varphi)$ for $\forall g \in G$. We say that G igllable if there is an invariant babaj on $\mathcal{L}(G)$.

Let $\Gamma_K = (V,E)$, $K = (A,B)$ is a K-hypergraph. Given a set $T \subseteq V$, let $|T|^{\Theta} = \sum_{v \in T} |\mathcal{S}t(v_1)|^{\Theta}$. Say that a transitive graph Γ_K is Θ-*igllable* if for $\forall \varepsilon > 0$, $\varepsilon \in (0,1)$, there is $T \subseteq V$, such that $|\delta_V^T|^{\Theta} < \varepsilon |D|^{\Theta}$.

Let $\ell^{\infty}(V)$ an infinite space of measurable real-valued functions on the vertex V of the K-hypergraph $\Gamma_K = (V,E)$, $K = (A,B)$. A babaj on $\ell^{\infty}(V)$ is called invariant if $\forall \varphi \in \ell^{\infty}(V)$ has the babaj as $\mathcal{L}_g(\varphi)$ with defines as function taking $v \mapsto \varphi(g(v))$ for $\forall g \in \mathcal{A}ut(\Gamma_K)$.

Theorem 10. Let $\Gamma_K = (V,E)$, $K = (A,B)$ is a transitive K-hypergraph. Then Γ_K is Θ-igllable $\Leftrightarrow$ $\mathcal{A}ut(\Gamma_K)$ is Θ-igllable and unimodular.

If Γ_K is β-crownable, then this concept is the same as ε-igllable.

The proof of theorems 8–10 and computational algorithms are in [4] and [7].

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Хаотики и их применения в биовычислениях

Идея «комбинаторного хаотика» была первоначально предложена В. В. Грицаком-Грёнером в его новаторской статье [1]. В настоящей статье мы строим модель комбинаторного хаотика из набора объектов биоинформатики:

a) потоки в хаосе;

b) хаос в перестановках;

c) хаос в комбинаторных конфигурациях.

Мы также строим вычислительные алгоритмы проблем a)-c).

Ключевые слова: хаос, синергетика, биоинформатика, алгоритм хаоса.