SYNERGETICS AND THEORY OF CHAOS

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MATHEMATICAL ASPECTS OF CHAOS

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We study the pattern recognition algorithms of combinatorial chaos (chaotic). First chaotic introduced in [1]. There are the most universal mathematical construction of chaos and, contrary to all the others, expand the notion of chaos even to finite structures. We give a mathematically exact characterization of chaos in finite sets. For example, a class of chaotic, that is "whirligig" and corresponds to granular chaotic structures, is proposed. The examples of recognition of minimum by the amount of elements among all the others (whirligig, anthill, disorder and quasimatroid etc) are given.

Keywords: chaos, algorithm, matroid.

Реферат

Точное и всеобъемлющее определение понятия хаоса — одна из наибольших проблем математической физики и в целом математики XX века (см. [3] и [4]), так и перешедшая нерешенной в XXI век. Модное понятие *фрактальности* (*фрагментарности*, как это называлось первым автором — А. Мальцевым в 1929 г.) не является определением хаотического объекта или процесса. Большинство известных фракталов (например, треугольник Жордана или ковер Серпинского) не хаотичны, так как обладают четко определенным структурным (не хаотическим!) строением, см. рис. 1. А в случае конечных объектов вообще неприменимы, поскольку в определении фрактальной размерности, являющейся единственным критерием распознавания фрактальности, необходим двукратный предельный переход, который, в свою очередь, требует бесконечного числа элементов.

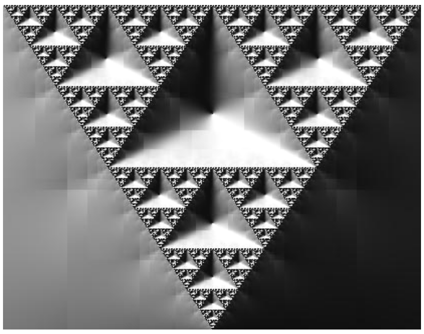


Рис. 1. Фигура и линия дробной размерности: ковер Серпинского.

Вообще физическая ценность так называемых фракталов — в их дробной *фрактальной размерности*, а не в том, что они могут служить примерами хаотичности. Хотя необходимо отметить, что фрактальная размерность не является пространственной характеристикой. Поэтому популярный «критерий хаотичности» объекта O: $|\dim(O) - \mathrm{dih}(O)| > 0$, где dim(O) — топологическая размерность (пространственная!), а dih — фрактальная размерность (не пространственная!), всего лишь пример анекдотической формулы, где яблоки вычитаются из стульев. Объекты с дробной пространственной размерностью существуют (скажем, примеры Понтрягина) и их следует распознавать непосредственным вычислением топологической размерности. Это представляет очень интересную проблему новейшей физической науки — «компьютерной физики». В конце концов, у всех классических фракталов нет пространственной дробной размерности. Например, треугольник Жордана и ковер Серпинского обладают размерностью 0.

Как заметили все авторы, перешедшие к фрактальным методикам, стохастические и вероятностные объекты и процессы также не адекватны хаотическим. Причина неадекватности —

обязательное выполнение закона аддитивности (аксиома теории вероятности!). В нашем подходе стохастический объект может быть хаотическим только в специальных вырожденных случаях, когда аксиома аддитивности становится несущественной. Например хаотичен, при равномерном кластерном разбиении объекта (см. рис. 2).

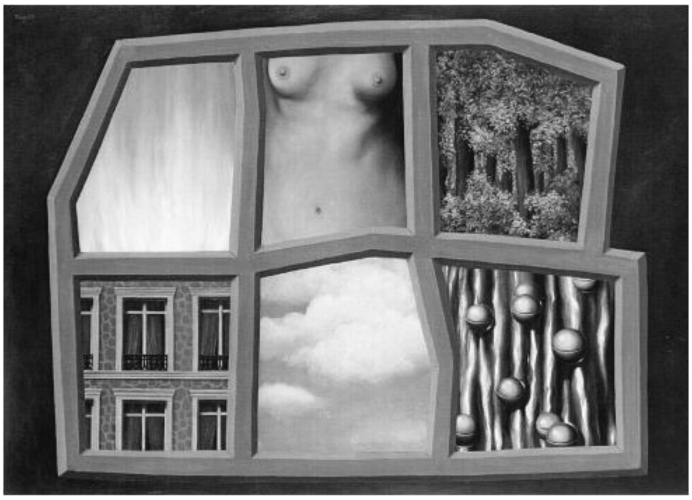


Рис. 2. Пример равномерного кластерного разбиения — Р. Магритт, «Окно».

С другой стороны, любой читатель нашей статьи с абсолютной точностью скажет, что, например, целочисленный конечный колчан:

0 7 0 7 0 7 0 7 0 7 0 7 0 7 0 7 0 7 0 7 (А)

не является хаотичным. А колчан:

3 7 0 1 1 4 5 2 7 7 0 3 3 3 3 4 6 7 5 1 (Б),

скорее всего, хаотичен. Разработанная нами теория полностью подтверждает интуицию наших читателей: колчан (Б) — хаотичный, а колчан (А) не хаотичный.

Более того, мы предлагаем алгоритмы расчета числовых характеристик их хаотичности (существует теория, также разработанная нами, которая рассчитывает степень структурированности, например, колчана (А), но мы ее предложим в последующих наших статьях и книгах).

Идея построения хаотичных структур базируется на фундаментальном факте оснований современной математики (а также топосной теории), гласящем, что любая структура на произвольном множестве определяется своим *классификатором* [5]. В случае булиана, чего достаточно для всей классической математики и для большинства физических применений, классификатор определяет для любой пары множеств $\mathfrak{A}$ и $\mathfrak{B}$, принадлежащих исследуемому пространству $\mathfrak{I}$, выполняется ли:

$$\mathfrak{A} \subseteq \mathfrak{B} \text{ либо } \mathfrak{B} \subseteq \mathfrak{A} \text{ ? } (*)$$

Отсюда очевидный вывод:

Если для пространства $\mathfrak{I}$ и для произвольной пары его подмножеств $\mathfrak{A}$ и $\mathfrak{B}$ вложения (*) не выполняются, то $\mathfrak{I}$ не обладает никакой математической структурой, или, иными словами, имеет хаотическую структуру.

Пространства, обладающие хаотической структурой, мы называем *хаотиками*. На протяжении этого научного поиска мы исследуем только конечные хаотики, которые мы называем *комбинаторными хаотиками*. (Принцип (*) для бесконечных хаотиков также выполняется и является единственным. Но нас не удовлетворяет современная теория конечности, исследования которой мы начнем в последующих изысканиях, результатом которых станет объяснение парадоксальных результатов вычислительных экспериментов Майорова в статистической физике). Для комбинаторных хаотиков удается получить более-менее детальную классификацию, см. рис. 9, с интересной интерпретацией. Например, хаотики под названием *юла* обладают крупнозернистым строением, а *муравейники* (anthill) имеют мелкозернистое строение. Очень интересны своей экстремальностью комбинаторные хаотики, называемые *матроидами*. Матроиды — это предельно возможные хаотические структуры. Именно на этом свойстве матроидов основываются наши алгоритмы определения *меры хаотичности*, равной минимальному числу элементарных итераций минимального алгоритма преобразования исследуемого хаотика в матроид. Построение реальных программных реализаций таких алгоритмов — ближайшая, очень важная в применениях проблема наших коллег — компьютерных физиков.

1. Introduction

A mathematical simulating natural process by deterministic dynamical biological systems requires usually simplifying approximations and assumptions. All classical models of statistical physics, for example: Gibson sums require infinite quantity of members, at least, denumerable or continuum. In the physical realty we have always only finite, in order 10^{23} (Avogadro number) quantity of elements and biologic 10^{12} (~number cells in man!).

In this point we give a formal (i. e. mathematically exact) characterization of chaos. In biology applications we often come across the so-called Chaotic Models.

Example 1. Forest, glass, floack.

For the problems of the classification chaotic structures we can manage without continual approximation ("continuous paradise") of the ground set model.

In the majority of publications the object O is called chaotic or fractal if:

$$|\dim(O) - \mathrm{dih}(O)| > 0, \qquad (1)$$

where dim(O), dih(O), respectively, topological and Hausdorf dimensions of the object O. Fractal definitions (1) of chaos hat whole series of lacks. There are:

1. The strict definition and calculation dih(O) need infinite numbers of elements O.
2. dim(O) also can be nonintegeral and then (1) absolutely ineffective.

The second face of fractality is self-similarity, which indicate that:

fractals have the inner structure!

Chaos provides a groundset for randomness and all nondeterminism's model. In such cases we can't make predications that are far better than those we would except from a traditional model based on the theory of random processes and other continues methods.

Therefore, if A is a set, and 2^A is the set of all subset of A, then chaotic $\Im$ is set of elements 2^A, for which no exist one pair, which is ordered by inclusion of subsets A.

Chaotic as a finite combinatorial chaos model was considered first in 1989 (detailed see [1]).

2. Simplest Recognition Algorithm of Strength Chaotic Height

At a basic level, one would expect that biological chaos would address the question of how one constructs the chaotic height for fundamental objects in biology. As usual, biological fundamental objects (the amino acid sequences of proteins, the neuron sequences) are strength of bio-objects (b-objects). Any such list of objects gives the objects of a linear order. These mean that if α and β are b-objects, then we can say that $\alpha < \beta$ if α precedes β on the list. A structure different from linear order early gives a chaotic combinatorial configuration on b-objects. If the list has n b-objects, the linear order must map these objects to the set 1, 2, 3, …, n. Further- more, one might expect that the linear order have chaotic height of strength (**chs**) null.

Definition 1. A ***permutation*** of n distinct b-objects of length i is an ordered arrangement of any i of the b-objects and is denoted Π_n^i. By P(n) denote the permutation of n distinct b-objects of length n. A permutation P(n) is frequently called a permutation of n.

Example 2. For example, the permutation of $S_4 = \{\alpha,\beta,\chi,\delta\}$ of length 2 are $\alpha\beta$, $\alpha\chi$, $\alpha\delta$, $\beta\alpha$, $\beta\chi$, $\beta\delta$, $\chi\alpha$, $\chi\beta$, $\chi\delta$, $\delta\alpha$, $\delta\beta$, and $\delta\chi$.

Theorem 1. The number of permutations of n b-objects of length i is

$$\mu(P(n)) = n \times (n-1) \times \ldots \times (n-i+1).$$

The proof is given in exercises.

Definition 2. Let P(n) is a permutation n. A ***strength chaotic height*** (**sch** is denoted $\lambda(P(n))$ of P(n) is equal to $\lambda(P'(n-1))\times n + (i_0-1)$ if n is odd number and **sch** is equal to $\lambda(P'(n-1))\times n + (n-i_0)$ if n is even number, where P'(n–1) is a permutation n–1 is equal to P(n) without n and i_0 is position n in P(n). $\lambda(P(1)) = 0$.

A ***relative strength chaotic height*** (**rsch** is denoted $\chi(P(n)))$ of P(n) is equal to $\lambda(P(n) : n!$, where $n! = 1 \times 2 \times \ldots \times n$.

Example 3. For example, we computation $\lambda((2,3,1,5,4))$ and $\chi((2,3,1,5,4))$. We have $\lambda((1)) = 0$. Therefore, we have

$$\lambda((1)) = 0 \Rightarrow \qquad \lambda((2,1)) = 0\times 2 + (2-1) = 1;$$

$$\lambda((2,1)) = 1 \Rightarrow \qquad \lambda((2,3,1)) = 1\times 3 + (3-2) = 4;$$

$$\lambda((2,3,1)) = 4 \Rightarrow \qquad \lambda((2,3,1,4)) = 4\times 4 + (4-4) = 16;$$

$$\lambda((2,3,1,4)) = 16 \Rightarrow \qquad \lambda((2,3,1,5,4)) = 16\times 5 + (4-1) = 83.$$

Finally, we obtain $\chi((2,3,1,5,4)) = 83/120 \approx 0.69167$.

Theorem 2. If P(n) is a permutation n, then

$$0 \le \lambda(P(n)) < n!.$$

The proof is trivial.

Corollary. If P(n) is a permutation n, then

$$0 \le \chi(P(n)) < 1.$$

The **sch** and **rsch** algorithm is listed in Algorithm 1. If necessary operate with a large numbers, then see an algorithm in [1], [2].

```
/******************** Algorithm 1. **********************************
challenge:
#include "heiperm.cpp"
void main()
{
 int perm[6]={0,2,3,1,5,4};
 int lambda=hperm(5,perm);
 double hi=rhperm(5,perm);
};
*****************************************************/
int hperm(int n, int *perm)
// Commentary 1
{
 if (n==1) return 0;
 else
 {
  int* perm1; // Commentary 2
  int j;
  int i0; // Commentary 3
  perm1=new int[n]; // Commentary 4
  j=1;
  for (int i=1; i<=n; i++)
  {
   if (perm[i]!=n)
```

```
    {
     perm1[j]=perm[i]; j++;
    }
    else i0=i;
   };
   int lpn1=hperm(n-1,perm1); // Commentary 5
   delete [] perm1; // Commentary 6
   if ((n%2)==1) // Commentary 7
   return lpn1*n+(i0-1);
   else return lpn1*n+(n-i0);
  };
};

double rhperm(int n, int *perm)
// Commentary 8
{
 long f=1;
 for (long i=1; i<=n; i++) f=f*i; // Commentary 9
 double hp=hperm(n,perm);
 return hp/f;
};
```

3. Logical Realization of 2LCA

We present here a formal definitions of a general combinatorial configurations (cc) and special case cc — chaotic, independence systems.

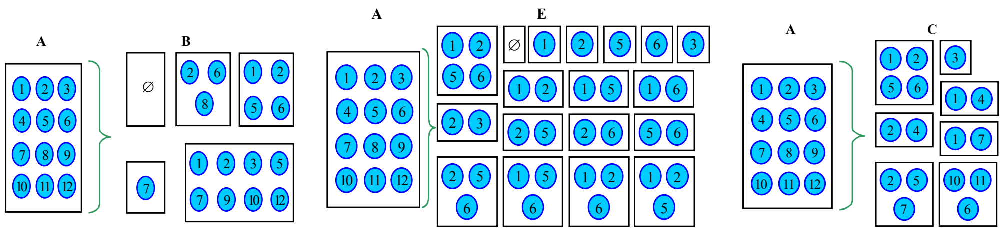
A Combinatorial Configuration K is an ordered pair

$$K = (A, B), B \subseteq 2^A$$

where A is a nonempty set. A is called the groundset of K, and elements of A are called the points of K. The elements of B are called the configurations of K.

Example 4. (See figure 1). A={1, 2, . . . , 12}, and

B={{∅}, {2, 6, 8}, {1, 2, 5, 6}, {7}, {1, 2, 3, 5, 7, 9, 10, 12}}, then K = (A, B) is a combinatorial configurations.



Figures 1-3.

Let I = (A, E), $E \subseteq 2^A$ is a combinatorial configurations. C. c. I is called *independece systems* if

$$(N)\ e_1 \in E, \text{ then } \forall e_2 \subseteq e_1 \Rightarrow e_2 \in E.$$

The elements of E are called the independence set of I.

Example 5. (See figure 2) A={1, 2, . . . , 12}, and E = {{∅}, {1,2,5, 6}, . . .{1, 2, 5}}. The pair I = (A, E) is an independence system.

Let $\Im = (A, C)$, $C \subseteq 2^A$ is a combinatorial configurations. $\Im$ is called *orderless systems* or *chaotic* if

$$(H)\ c_1, c_2 \in B\ \&\ c_1 \subseteq c_2 \Rightarrow c_1 = c_2.$$

The elements of C are called the *cycle* of $\Im$.

Example 6. (See figure 3) A={1, 2, . . . , 12}, and C = {{1,2,5,6}, . . ., {10,11,6}}. The pair $\Im$ = (A,C) is a chaotic.

Example 7. Let A are a pack of wolves and C are all coalition. The pair $\Im$ = (A,C) is a chaotic.

Example 8. Let A are a swarm of bees and E are all subspecialisation. The pair V = (A, E) is an independence system.

4. Chaotic Construction

Let $\Im$ = (A,C) be a chaotic. A set $-(B)_{\Im} \subseteq A$ is called a *closure* of a subset $B \subseteq A$ if the following conditions hold:

1) $B \subseteq -(B)_{\Im}$,
2) $b \in -(B)_{\Im} \setminus B \Leftrightarrow$ there exists two f-sequences $\{\alpha(i)\}$, $\{\tau(i)\}$, where $\alpha(i) \in A$, $\tau(i) \in C$, $\alpha(i) \neq \alpha(k)$, $\alpha(n) = b$, $\tau(i) \neq \tau(k)$ such that $\alpha(i) \in \tau(i) \subseteq B \cup (\bigcup_{1}^{i} \alpha(j))$, where $1 \leq i < k \leq n$, j=[1,i], i = [1,n].

By $-(B)_{\Im}$ denote a *closer* of a subset $B \subseteq A$ in $\Im$. A subset $D \subseteq A$ is called a *flat* if $-(D)_{\Im} = D$. A minimal flat is called an *atom*. A flat $P \subseteq A$, $\mu(P) = 1$ is called a *loop*.

Example 9. (See figure 4). A={1,2, . . . ,8}, Ω={{1,2,3,4}, {1,2,5,6}, {5,6,7,8}}. $-(\{1,2,3\})_{\Im} = \{1,2,3,4\}$.

Let $\Im$ = (A,C) be a chaotic. A subset $N \subseteq A$ is called *independent* in $\Im$ if $\forall \chi \in C$, $\chi \setminus N \neq \varnothing$. A subset $S \subseteq A$ is a *spanning* subset of $\Im$ if $-(S)_{\Im} = A$. Minimal spanning subsets $B \subseteq A$ are *bases* of $\Im$. A maximal nospannig flat $K \subseteq A$ is a *coatom* of $\Im$. Con-sider the family $\Xi(\Im)$ of all bases of $\Im$; then this family is called bases system of $\Im$. Consider the family $\Theta(\Im)$ of all flats $\Im$. Consider the family T($\Im$) of all atoms of $\Im$; then this family is called body of $\Im$. Also, consider the collection Ħ($\Im$) of all maximal are no spanning subsets in $\Im$; then this collection is called the *interior systems* of $\Im$.

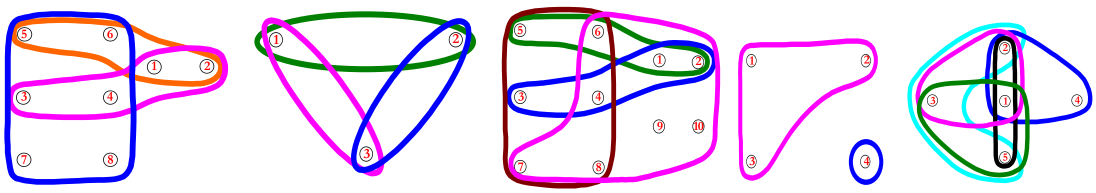


Figure 4-8.

Example 10. (see figure 4). For example, a independent set N={4, 8}. S = {1, 2, 3, 4, 6, 7, 8} is the spanning subset and no base. B = {1, 2, 4, 6, 7, 8} is the base. H = {1, 2, 3, 4, 5, 6} is the hyperflat.

Let $\Im$ = (A,C) be a chaotic and $a \in A$ be an element of A. The element a is a loop if $a \notin \chi$ for $\chi \in C$. And the element b is a coloop if $b \in \chi$ for $\chi \in C$. The chaotic obtained by *separation* of a, denoted by $\Im \setminus a$, is the chaotic:

1) $(A \setminus \{a\}, \chi: a \notin \chi \in C)$ if a is not a coloop of $\Im$,

2) $(A\backslash\{a\}, \chi\backslash\{a\}: \chi \in C)$ if a is a coloop of $\Im$.

The chaotic obtained by *detachtion* of a, denoted by $\Im/a$, is the chaotic:

3) $(A\backslash\{a\}, \chi\backslash\{a\}: a \in \chi \in C)$ if a is not a loop of $\Im$,

4) $(A\backslash\{a\}, \chi: \chi \in C)$ if a is a loop of $\Im$.

A minor of $\Im$ is a chaotic m-$\Im$ that can be obtained from $\Im$ by a sequence of separations and detachtions.

A useful further specialization of these definitions is provided by

Let $\Im = (A,C)$ be a chaotic, $C = \{c_i \mid c_i \subseteq A, I = [1,n]\}$.

A pair $\Im^\circ = (A,C^\circ)$ be a complement of $\Im$ if $C^\circ = \{ \overset{\circ}{c_i} \mid \overset{\circ}{c_i} = A / c_i, i = [1,n]\}$.

Denote by $\beta_0(C)$ the family of all minimal subsets B of A such that $B \cap c_i \neq \varnothing$, $i = [1,n]$. We shall say that a map $\beta_0 : C \longrightarrow \beta_0(C)$ is called a block operation and the pair $\beta_0(\Im) = (A, \beta_0(C))$ is called a blocker of $\Im$.

Denote by $\beta_d(\Im)$ the family of all minimal subsets B of A such that $B \cap c_i \neq \varnothing$, $i=[1,n]$, where $0 \le d \le \mu(A)$ and $\mu(B) \ge d$. We shall say that a map $\beta_d : C \longrightarrow \beta_d(C)$ is called a d-*block operation* and the pair $\beta_d(\Im) = (A, \beta_d(C))$ is called a d-*blocker* of $\Im$.

We say that a denor of $\Im$ is a chaotic d-$\Im$ that can be obtained from $\Im$ by a sequence of d-block operations. An inor of $\Im$ is a chaotic i-$\Im$ that can be obtained from $\Im$ by a sequence of i-block operations, where $0 \le i \le \mu(A)$.

Now we shall give the following propositions.

Theorem 3. Let $\Im = (A,C)$ is a chaotic, $C=\{c_i \mid c_i \subseteq A, I = [1,n]\}$. Then the complement $\Im^\circ = (A,C^\circ)$ be the chaotic, where $C^\circ = \{ \overset{\circ}{c_i} \mid \overset{\circ}{c_i} = A/c_i, i= [1,n]\}$.

Theorem 4. If $\Im = (A,C)$ is the chaotic, then the d-blocker $\beta_d(\Im) = (A, \beta_d(C))$ is a chaotic, where $0 \le d \le \mu(A)$.

Theorem 5. If $\Im = (A,C)$ is the chaotic, then

$$\beta_0(\beta_0(\Im)) = \Im. \quad (2)$$

Theorem 6. If $\Im = (A,C)$ is the chaotic defined in definition 0.2.7 and $0 \le d \le \mu(A)$, then

$$\beta_d^{\nu_d+\rho_d}(\Im) = \beta_d^{\nu_d}(\Im), \quad (3)$$

where $Z^+ \ni \nu_d \ge 0$ and $Z^+ \ni \rho_d \ge 1$. ν_d is called a d-tail of $\Im$. ρ_d is called a d-period of $\Im$.

The proof is found in [1].

Theorem 7. If $\Im = (A,C)$ is the chaotic and $I(\Im) \subset C$ is the set of all independence subsets of $\Im$, then

$$I(\Im) = (\beta_0(\Im))^\circ. \quad (4)$$

5. Types of Chaotic

The main problem and hardcore of chaos theory is the classification of chaotic types of sets. Here the general idea of classification is attach combinatorial invariants, which may by algorithms. The ideal would be to have a combinatorial invariant which actually characterizes a chaotic type of polynomial algorithms.

It can be shown in the usual way that a matroid are limit's chaotic between an order and a chaos.

Let M = (A,Ω) be a chaotic and $\forall\alpha,\beta\in\Omega$, $\alpha\cap\beta\neq\varnothing$ are cycles of M. We say that the chaotic M is a matroid if $\forall a\in\alpha\cap\beta\Rightarrow\exists\chi\in\Omega$ and $\chi\subseteq(\alpha\cap\beta)\backslash\{a\}$.

Example 11. (see figure 5). A={1,2,3}, Ω = {{1,2}, {2,3}, {1,3,}}. M = (A,Ω) is the matroid.

Example 12. Let A = (A,Ω) be a chaotic. A chaotic A is an atomistic if for every a close set $B\subseteq A$ there exists the sets of atoms $A\ni\alpha_i$, $i\in I$ such that

$$-\left(\bigcup_{i\in I}\alpha_i\right)_A = B \qquad (5)$$

Example 13. A prokaryote P = (A,Ω) is atomistic chaotic, where A is the body of P, the cycles Ω are organs of the prokaryote P, every an atom $\alpha\in A$ of the chaotic P is a cell of the prokaryote.

Let J = (A,Ω) be an atomistic chaotic. An the chaotic J is a *jula* (jula-chaotic) if for every independent set E ($E\subseteq A$) of atoms of J there exists an atom $\sigma\subseteq A$ and coa-tom $\tau\subseteq A$ such that $-\left(\bigcup_{\alpha_i\in E/\sigma}\alpha_i\right)_J\subseteq\tau$, where α_i are the atoms of J but $\sigma\not\subset\tau$.

Example 14. (see figure 6). A={1,. . .,10}, Ω = {{1,2,3,4}, {1,2,5,6}, {3,4,5,6,7,8}, {1,2,4,6,7,8,9,10}}. J = (A,Ω) is the jula.

Let F = (A,Ω) be an atomistic chaotic. An the chaotic F is a *dust* if for every independent set E ($E\subseteq A$) there exists an atom $\sigma\subseteq A$ and coatom $\tau\subseteq A$ such that $-\left(\bigcup_{\alpha_i\in E/\sigma}\alpha_i\right)_F\subseteq\tau$, where α_i are the atoms of F but $\sigma\not\subset\tau$.

Let H = (A,Ω) be an atomistic chaotic. By T_α denote the set of atoms $\alpha^*\in T(H)\ni\alpha$ such that we have $\alpha^*\subset\alpha$. An the chaotic H is a *anthill* if for every a close set B ($B\subseteq A$) and for every D ($A\supset D\not\subset T_B$) such that $-\left(\bigcup_{i\in I}\alpha_i\right)_H = D$, where α_i are the atoms there is an atom $\alpha^B\in T_B\backslash D$ and an independent subset E of D such that $E\cup\{\alpha^B\}$ is not independent.

Example 15. (see figure 7). A={1,2,3,4}, Ω = {{1,2,3}, {4}}. H = (A,Ω) is the anthill.

Let D = (A,Ω) be an atomistic chaotic. By T_α denote the set of atoms $\alpha^*\in T(D)\ni\alpha$ such that we have $\alpha^*\subset\alpha$. An the chaotic D is a fog if for every close set B ($B\subseteq A$) and for every D ($A\supset D\not\subset T_B$) such that $-\left(\bigcup_{i\in I}\alpha_i\right)_D = D$, where α_i are the atoms there are the atoms ${}^B\alpha_i\in T_B\backslash D$ and an independent subsets E_i of D such that $E_i\cup\{\alpha^B\}$ are not independent.

Let K = (A,Ω) be an atomistic chaotic. An the chaotic K is a *quasimatroid* if K is both a jula and an anthill.

Example 16. (see figure 8). A={1,. . .,5}, Ω = {{1,2,3}, {1,2,4},{1,2,5}, {1,3,5}, {2,3,5}}. K = (A,Ω) is the quasimatroid.

Let $L = (A,\Omega)$ be an atomistic chaotic. An the chaotic L is a *formless* if L is not both a jula and an anthill.

Example 17. (See figure 4). $A=\{1,2, . . . ,8\}$, $\Omega=\{\{1,2,3,4\}, \{1,2,5,6\}, \{5,6,7,8\}\}$. $L=(A,\Omega)$ is the formless.

We may assume that a julies are modell of granular chaos and an anthills are mo-dell of uniformly chaos. These results can be summarized as follows.

Theorem 8. $J=(A,\Omega)$ be the jula defined in definition 0.2.10. If J is have not a loop B, then of all cycle $\alpha \in C$

$$\mu(\alpha) \geq 3. \tag{6}$$

The proofs are found in [1].

Corollary 6.1. Suppose $J=(A,\Omega)$, $\mu(A)<\infty$ be an atomistic chaotic; then there exists a polynomial complex algorithm of to see that J be a jula.

Theorem 9. Let $H=(A,C)$ be an atomistic chaotic and $B \subseteq A$ be an arbitrary independence subset of H. H is an anthill iff there exists an element $b\in B$ such that

$$b \notin (B/\{b\})^{\circ} \tag{7}$$

The proofs are found in [2] or [1].

Corollary 9.1. Suppose $H=(A,C)$, $\mu(A) < \infty$ be an atomistic chaotic; then there exists a polynomial complex algorithm of to see that H be an anthill.

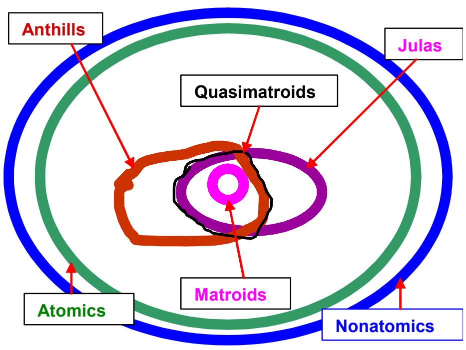


Figure 9. Types of Chaotic.

Some other classifications of chaotic types will be given after blocking methods.

The semimatroid is close to matroids. Let $P=(A,\Omega)$ be a chaotic. A chaotic P is a *semimatroid* if $\beta_1(P) = P$, where β_1 is the 1-block operation.

Theorem 10. Let $P=(A,\Omega)$ be a chaotic and $B \subseteq A$ be an arbitrary independence subset of P. P is a semimatroid iff there exists an element $b\in B$ and a hyperflat $\Gamma\subseteq A$ such that

$$\Gamma \cap B = B /\{b\} \tag{8}$$

The proof is found in [4–6].

Corollary 10.1. Suppose $P=(A,\Omega)$, $\mu(A)<\infty$ be a chaotic; then there exists a polynomial complex algorithm of to see that P be a semimatroid.

Theorem 11. Let M = (A,Ω) be a chaotic. A chaotic M is a matroid iff

$$\beta_1(M) = \beta_0(\beta_0 (M)°), \quad (9)$$

where β_0 and β_1 are the block operation and the 1-block operation.

Corollary 11.1. Suppose M = (A,Ω), $\mu(A)<\infty$ be a chaotic; then there exists a polynomial complex algorithm of to see that M be a matroid.

6. Algorithms of Recognition of Degree of Chaotic

Many problems in physics, biology and medicine involve determining the measure of disorder of several objects. For instance, finding that structure which computed chaotic degree can approach determining the disorderly structure of swelling or tumor. We introduced the concept of x-degree (Θ-simple degree) as a measure of disorder in a chaotic. We given the notion of x-degree and Θ-simple) degree algorithm schemes. These algorithms are discussed in more detail in [2].

Let $\Im = (A,C)$ be a chaotic. By $dimh(\Im)\in Z^+$ denote a degree of disorder $\Im$. If $\Im$ is a matroid, then $dimh(\Im) = 0$. Let $\Im^* = (A^*,C^*)$ is an other chaotic. Further, $a\in A$ and $a_1 \in A^*$. We can assume that

$$\left| dimh(\Im) - dimh(\Im^*) \right| = 1 \quad (10)$$

if the following conditions hold:

1) $= \Im^* \setminus a_1$,
2) $= \Im^* / a_1$,
3) $\Im^* = \Im \setminus a$,
4) $\Im^* = \Im / a$.

The chaotics $\Im$ and $\Im^*$ are called *adjacent*.

A x-*walk* is a sequence L:

$$\Im_1 \Im_2 \ldots \Im_n \quad (11)$$

of chaotics $\Im_i$, i =[1,n], in which $\Im_i$ and $\Im_{i+1}$ are adjacent, $\Im_1$ is the input of L and $\Im_n$ is the outcome of L. We say that the chaotic $\Im_1$ has x-degree n, if $\Im_n$ is a matroid and L is a minimal x-walk. Moreover, we say that the chaotic $\Im_1$ is called Θ-simple degree n, if $\Im_n$ is Θ and L is a minimal x-walk if the following conditions hold:

a) is an atomic;
b) is a formless;
c) is a fog;
d) is a dust;
e) is a jula;
f) is an anthill;
g) is a quasimatroids.

Theorem 12. Let M = (A,Ω), A ≠∅, H_1 =(A_1,{(1,2), (1,3)}), H_2 =(A_2,{(1,2,3)}) are a chaotics. A chaotic M is a matroid iff M has not be x-walk L= $\Im_1 \Im_2 \ldots \Im_n$ (11), where $\Im_1$ = M and $\Im_n = H_1$ or $\Im_n = H_1$.

Corollary 12.1. Suppose M = (A,Ω), $0<\mu(A)<\infty$ be a chaotic; then there exists a polynomial complex algorithm of to see that M be a matroid.

The proof is found in [2].

Further, in theorem13, we proved that the algorithm X1 is finite.

Theorem 13. Proposition 0.2.11. Let $M = (A,\Omega)$, $\mu(A) <\infty$, is a chaotic. M has be a finite x-walk $L= \Im_1 \Im_2 . . . \Im_n$, where $\Im_1 = M$ and $\Im_n = M_0$, where

$$M_0 = (A_1,\{(\varnothing)\}) \tag{12}$$

is the homogeneous matroid.

x-degree degree algorithm scheme X1

Input: a chaotic $\Im = (A,\Omega)$.

Compute: $\Im_1 \Im_2 . . . \Im_n$ — a minimal x-way, where $\Im_n$ is a matroid.

Output: the x-degree of a chaotic $\Im$ is equal n.

A Θ-degree degree algorithm scheme (XΘ) of a chaotic is defined similarly. The listings of the algorithms X1 and XΘ are given in additional of the book [6].

Let $\Im = (A,C)$ be a chaotic. By $\dim\beta(\Im)\in Z^+$ denote a β-degree of disorder $\Im$. If $\Im$ is a matroid, then $\dim\beta(\Im) = 0$. Let $\Im^\beta = (A,C^\beta)$ is an other chaotic. Further, $a\in A$ and $a_1\in A^*$. We can assume that

$$|\dim\beta(\Im) - \dim\beta(\Im^\beta)| = 1, \tag{13}$$

if the following conditions hold:

a) $\Im = \beta_d(\Im^\beta) = (A, \beta_d(C^\beta))$, $1 \le d \le \mu(A)$,

b) $\beta_d (\Im) = \Im^\beta$, $1 \le d \le \mu(A)$.

The chaotics $\Im$ and $\Im^\beta$ are called β-*adjacent*.

To complete the proof of $\Im$ and $\Im^\beta$ are chaotics, we use of theorem 12.

A β-*walk* is a sequence L:

$$\Im_1 \Im_2 . . . \Im_m \tag{14}$$

of chaotics $\Im_i$, i =[1,m], in which $\Im_i$ and $\Im_{i+1}$ are β-adjacent, $\Im_1$ is the input of L and $\Im_m$ is the outcome of L. We say that the chaotic $\Im_1$ has β-degree n, if $\Im_m$ is a matroid and L is a minimal β-walk. Moreover, we say that the chaotic $\Im_1$ is called Ξ-simple degree n, if $\Im_m$ is Ξ and L is a minimal β-walk if the following conditions hold:

a) is an atomic;
b) is a formless;
c) is a fog;
d) is a dust;
e) is a jula;
f) is an anthill;
g) is a quasimatroid;
h) is a semimatroid.

We are now in a position to state the problem of chaotic theory.

Problem 1. Let $H = (A,\Omega)$, $4< \mu(A) <\infty$ is a chaotic, $2 < \beta$. H has be a finite β-walk $L = \Im_1 \Im_2 . . . \Im_m$ (14), where $\Im_1 = H$ and $\Im_m = B$. Where the binor

$$B =(A, \Omega^*) \tag{15}$$

is a matroid?

A β-degree degree algorithm scheme Xβ of a chaotic is similarly X1. The listing of the algorithms Xβ is given in additional of the book [2].

β-degree degree algorithm scheme Xβ

Input: a chaotic $\Im = (A,\Omega)$.

Compute: $\Im_1 \Im_2 \ldots \Im_m$ — a minimal β-way, where the binor $\Im_m$ is a matroid.

Output: the β-degree of a chaotic $\Im$ is equal m, or we have the counterexample to problem 1.

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Математические аспекты хаоса

Изучаются алгоритмы распознавания комбинаторного хаоса (хаотиков). Впервые хаотик был введен в работе [1]. Это наиболее универсальная конструкция хаоса, которая, в отличие от других конструкций, применима и для конечных структур. Более того, мы впервые строго и точно характеризуем и классифицируем хаотические структуры для конечных множеств. Классификация представляет интерес для получения адекватного соответствия с реальными хаотическими структурами. Например, класс хаотиков, который соответствует «вихрям», адекватен гранулированным структурам. Для каждого хаотического класса (вихрь, муравейник, беспорядок, квазиматроид и др.), мы распознаем структуры с минимально возможным числом элементов.

Ключевые слова: хаос, алгоритм, матроид.